INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS B: ATOMIC, MOLECULAR AND OPTICAL PHYSICS

J. Phys. B: At. Mol. Opt. Phys. **36** (2003) 1049–1061 PII: S0953-4075(03)54733-2

Entanglement and inhibited quantum evolution

P E Toschek, Chr Balzer, Th Hannemann, Ch Wunderlich and W Neuhauser

Universität Hamburg, Institut für Laser-Physik, Jungiusstraße 9, D-20355 Hamburg, Germany

E-mail: toschek@physnet.uni-hamburg.de

Received 14 October 2002, in final form 20 January 2003
Published 24 February 2003
Online at stacks.iop.org/JPhysB/36/1049

Abstract
The evolution of a quantum system is impeded by the system's state being observed. A test on an ensemble neither proves the causal nexus nor discloses the nature of the inhibition. Two recent experiments that make use of sequential optical or microwave-optical double resonance on an *individual* trapped ion disprove a dynamical effect of back action by meter or environment. They rather indicate the ionic states involved in the evolution being entangled with the potentially recorded bivalued scattered-light signal.

1. Introduction

Although Erwin Schrödinger coined the term 'verschränkter Zustand' or 'entangled state', the prototypical representation is the pair of correlated spins of the EPR gedanken experiment [1]: the observation of one spin's orientation determines the direction of the distant second one. The principal author could not help but invoke—and ridicule—what he called 'spooky action on a distance'. Today we ascribe the odd consequence of entanglement to the non-locality of quantum physics.

Another example of entangled states is concerned with the different degrees of freedom of an ion in a trap. An electronic moment is radiatively excited on a spectral sideband that is generated by the phase modulation of the moment from the ion's centre-of-mass vibration: the respective electronic and centre-of-mass eigenstates become pairwise correlated, and electronic excitation arises along with the vibrational one. Whereas the supposed interaction seems less spooky since the involved distance—namely that between electron and nucleus—is so small, this example is most important since it provides the basis of quantum information processing on an array of trapped ions [2]. But what about a common situation of quantum measurement, where an individual quantum system, say a single ion, is radiatively prepared in a superposition of (meta-)stable eigenstates and sequentially probed by tentative excitation of resonance scattering out of one of these states [3]? According to the signal of the scattered light being found or missed, the quantum system is left over in the corresponding eigenstate [4]. Protagonists of measurement theory have claimed that the back action of the meter—or, perhaps, of the environment—is responsible for this 'state reduction', in the spirit of what has been called the 'Heisenberg microscope' [5]—although this presumptive back action, over macroscopic distances, seems spooky indeed.

A conspicuous manifestation emerges with *reiterated* measurement on a quantum system: such a repeated or continued observation must bring the quantum system's evolution to a halt [6, 7]. This somewhat odd issue, the 'quantum Zeno effect' (QZE) [8], was noticed decades ago. Even professed epistemological realists have ascribed the impediment or even the disappearance of the quantum evolution to a, more or less local, perturbation of the *specific quantum* evolution by the measuring device. They should have been cautioned by the well-known fact that the deterministic evolution of a *classical* quantity as the rotating polarization of a light beam that propagates in a sugar solution suffers analogous impediment by reiterated measurement (see, e.g., [9, 10]): placing more and more analyser plates—let their number be n—at the initial angle of polarization inside the beam reduces, by factor n, the net angle of rotation detected at the final analyser and even the projective loss vanishes. This modified evolution is easily modelled in the configuration space of light polarization which is the Poincaré sphere. Locally isomorphic with it is the Bloch sphere of $SU(2)$ symmetry that represents the configuration space of a quantum system made up of two-level atoms, or of spins [11, 12]. Accordingly, the modelling of a quantum ensemble and of its *random* evolution is quite similar, and the assumption of a dynamical back action of the meter seems as natural as the effect of the analyser filters on the light beam. However, arguments based on entanglement and the interpretation of quantum mechanics seem to admit or even demand the *absence* of dynamical interaction, a situation referred to as the 'quantum Zeno paradox' (QZP) [13]. Thus, the urgent question arose, whether or not impeded *quantum* evolution could be demonstrated, and if so, under conditions that *prevent* any back action on the quantum object to arouse this effect.

Recent work on the QZE has dealt with both fundamental problems and proposals for observations [14, 15].

A tricky experiment on the demonstration of the QZE was attempted more than a decade ago on 5000 ions stored in a trap [16, 17]. Its results completely agreed with the expectation values predicted from quantum mechanics. However, more recently it was shown that a quantum object consisting of an *ensemble* is not suitable for the desired demonstration of the quantum evolution being retarded or inhibited, let alone for discrimination between the above alternatives.

(1) The applied scheme involved repeated irradiation of the ion ensemble by 'probe' light, but only final read-out of the effect of probing, such that back-and-forth transitions were systematically missed, both in any one particular ion, and in pairs of ions. In short, retrieval was taken for survival [18].
(2) The applied strategy did not discriminate against dynamic back action on the quantum object [13].
(3) Measurements on an ensemble yield expectation values; they are almost deterministic, but do not reveal the micro-state of the prepared quantum object. Thus, both dephasing of the quantum state, and potential pre-observational 'state reduction' of member ions—be it caused by the environment, or by the back action of the meter—go unnoticed [19–21] (figure 2). In contrast, a quantum object that consists of a *single* entity does allow such a discrimination: its micro-state is revealed by the random results of series of real measurements (including read-out of the results) after identical preparation, such that its evolution from the start to the observation is well documented.

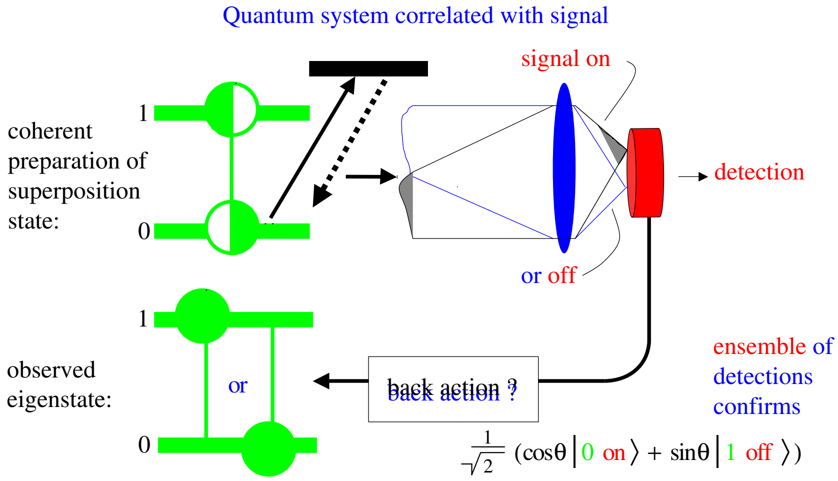

Figure 1. A single quantum system, prepared in a superposition of its lower state $|0\rangle$ and upper state $|1\rangle$ by coherent interaction with a resonant pulse of radiation of area θ (top left) does or does not scatter probe light such that 'signal *on*' or '*off*' is detected, respectively (top right). A schematically indicated beam of scattered probe light, marked by shaded sectors and 'signal on', correlates with the quantum system being in state $|0\rangle$. The other schematic beam, marked by 'off', and indicating null scattering, correlates with the quantum system being in $|1\rangle$. The system is left such that one of its eigenstates (bottom left) that is shared (0) or not shared (1) with the probe line. What kind of action, if any, has made the superposition become an eigenstate?

(This figure is in colour only in the electronic version)

Recently, two experiments on an *individual* Yb^+ ion have been performed that were, moreover, designed for discrimination against any kind of dynamic action upon the quantum object, and that meet the conditions required for an unequivocal demonstration of the QZE and even the QZP [22, 23].

2. Quantum evolution proved inhibited

In the recent experiments, the quantum object, a narrow resonance of the isolated ion, connecting the ground state with a metastable state, is driven by a resonant pulse of coherent radiation (length τ, Rabi frequency Ω) such that the ion, initially in its ground state, is assumed to become prepared in a predetermined superposition of the states that pertain to the resonance, with its state being placed at polar angle $\theta = \Omega\tau$ on the Bloch sphere. Immediately after this 'coherent' preparation the ion is probed by a pulse of laser light resonant with a neighbouring resonance line. Since the lower level of this line coincides with one of the levels of the driven resonance, the successful excitation of resonance scattering indicates that the ion will be found in the eigenstate of that common level. Absence of resonance scattering proves the ion to be found on the other, 'dark' level of the driven narrow resonance (figure 3). In fact, the found state is detected almost with certainty, i.e. the detection is deterministic, whereas the preparation of a *single* atom by the driving pulse is probabilistic. The probability of the ion surviving, after preparation and subsequent probing, in its *initial* state is

$$p_0 = \cos^2(\theta/2), \tag{1}$$

where relaxation as well as level degeneracy are neglected for the moment [24]. Now, let a second cycle of state preparation and probing be applied to the ion. If the first preparation

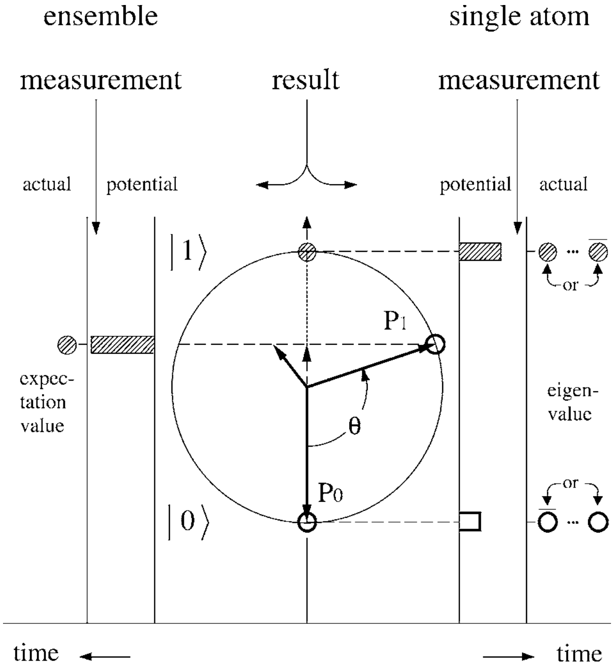

Figure 2. Preparation and measurement of two-level quantum system ($SU(2)$). P_0 initial state vector, P_1 after preparation, by pulse with area θ, of *single quantum system* (right). The expectation value from a measurement on an *ensemble* (left) may result from *any* P_1 state located at the horizontal plane indicated by the dashed line. Any such mixed state located inside the Bloch sphere would result from phase decoherence by interaction with measuring device, neighbour atoms, or environment. In contrast, a single-atom measurement makes use of a *pure* state P_1, located on the surface of the Bloch sphere, that results from the preparation $P_0 \rightarrow P_1$. The small bars indicate the probabilities of the results of measurement anticipated from θ; they are different, in general, for $|0\rangle$ and $|1\rangle$. The actual result is an eigenvalue: 1 (and not 0), or 0 (and not 1).

had yielded a 'real' superposition state, and the first probing had had *no* effect on that state of the quantum object, the quasi-spin that represents the driven resonance in configuration space would now be rotated by a total of *twice* the angle $\theta/2$. Otherwise, the effect of the probing 'makes the state set to an eigenstate'. This wording is tantamount to reconciling the result with having implicitly considered the preparative process deterministic in attributing reality to the prepared superposition state. This process is, however, random. The probability of the system's survival in its initial state is p_0 again, and after q subsequent cycles, the conditional probability is $V(q) = p_0^q$. Consequently, the likelihood of finding, in a long trajectory of measurements, q *identical results* in a series [22] is

$$U(q) = U(1)V(q-1). \tag{2}$$

Thus, a test for the effect of the probing on the ion simply requires the evaluation of the normalized distribution of *sequences of equal results*, $U(q)/U(1)$, in long enough trajectories of measurements, and comparison with the conditional probability of q-times survival, $V(q-1)$.

It is of prime importance, however, to verify the coherence of the interaction of driving radiation and ion in the course of the preparation. For this purpose, the transition rate, that is the normalized number of *on–off* and *off–on* pairs of results, is evaluated along a trajectory of

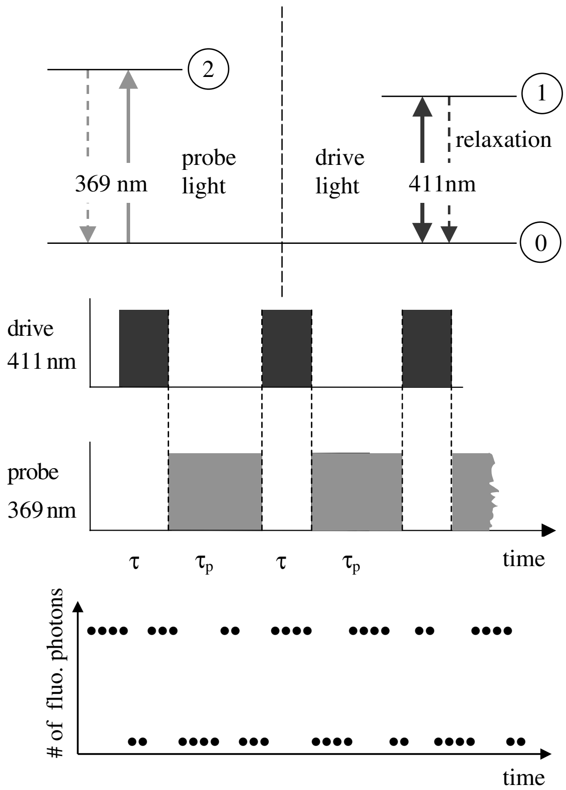

Figure 3. Alternating opto-optical double resonance: scheme of levels and excitation of $^{172}Yb^+$ (top). Temporal schedule (middle). Schematic trajectory of results (bottom).

measurements. With a long trajectory, the normalized rate closely approaches excitation and de-excitation probability, respectively. The values of this transition probability derived from trajectories recorded at stepwise incremented detuning of the driving radiation combine for an excitation spectrum across the respective line. This spectrum is to be compared with the probability for stimulated excitation [22],

$$p_{01} = \cos^2 \chi \, \sin^2(\theta/2), \tag{3}$$

where $\tan \chi = \Delta/\Omega$, $\theta = \sqrt{\Omega^2 + \Delta^2}\,\tau$, and $\Delta = \omega - \omega_0$ is the detuning of the driving radiation (ω) off its resonance ω_0. The appearance of fringes that modulate the spectrum with high visibility prove the coherence of the driving interaction.

3. Driving an electronic resonance of the ion

The scenario outlined above has been performed using narrow-band laser light of 411 nm wavelength for driving the $S_{1/2}$–$D_{5/2}$ electric quadrupole resonance of a single $^{172}Yb^+$ ion, and 369 nm light pulses for probing the ground state at the $S_{1/2}$–$P_{1/2}$ resonance line [22].

Trajectories of results of sequential measurements are made up of alternating 'waiting intervals' of random length. The transition probabilities evaluated from the number of waiting intervals—or, as outlined above, from the recorded number of pairs of consecutive unequal

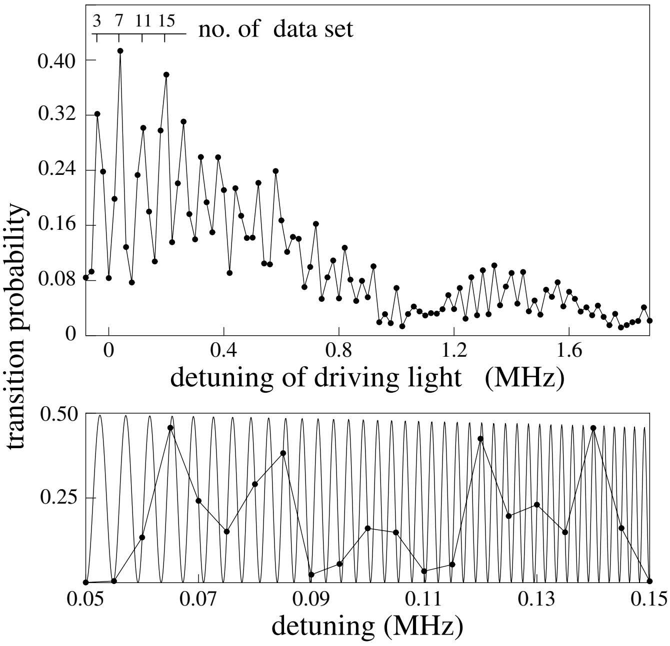

Figure 4. Probability of excitation, versus detuning $(\omega - \omega_0)/2\pi$ by 20 kHz steps of the drive (top). Note the first-order vibronic sideband at 1.3 MHz. Within a small range close to resonance, detuning may replace variation of the drive-pulse length τ. The spectrum of absorption is superimposed by stroboscopic sampling of the ion's Rabi notation, as demonstrated by a simulation with small steps on an expanded scale (bottom).

results—in the trajectories recorded at stepwise progressive detuning of the driving light is shown in figure 4. The dipole-forbidden line shows up substantially power-broadened. Also note the first vibrational sideband of the line some 1.3 MHz up from the carrier frequency. The characteristic modulation of some 80% visibility emerges as a result of stroboscopically sampling the transition probability modulated by the effective Rabi frequency θ/τ. This sampling has been numerically modelled and is shown, on an expanded tuning scale, at the bottom of figure 4.

The deep modulation of the spectrum of the transition probability proves the driving interaction to be coherent. Thus, the data, in particular the distributions of sequences of equal results embedded in the corresponding waiting intervals, match the principal precondition for applying equation (3) and, on resonance, equation (1) for the evaluation. It has turned out, however, that relaxation must be considered with a quantitative evaluation. This finding is somewhat surprising, since the rate of spontaneous decay on the driven resonance is very weak—the lifetime of the $D_{5/2}$ level is at least 5.7 ms [25]—and the bandwidth of the driving laser is found below 100 Hz, [22]. Moreover, the slightly non-degenerate Zeeman sublevels of the ion's ground state and of the $D_{5/2}$ excited level have to be taken into account, as well as the non-infinite length of the trajectories of the data. From a model of the light-driven ion dynamics extended in this way [22, 26], modified values of the—now differing—probabilities p_i of finding the ion in state $i = 0, 1$, correlated with signal '*off*' and '*on*', respectively, have been derived, inserted into $V(q-1)$, and made to fit in with the distributions of sequences of equal results, $U(q)/U(1)$ by variation of the pulse area θ and a parameter f_i. The data and the fits to $V(q)$ are shown in figure 5. The deviation of the parameter f_i from unity indicates redistribution among the Zeeman

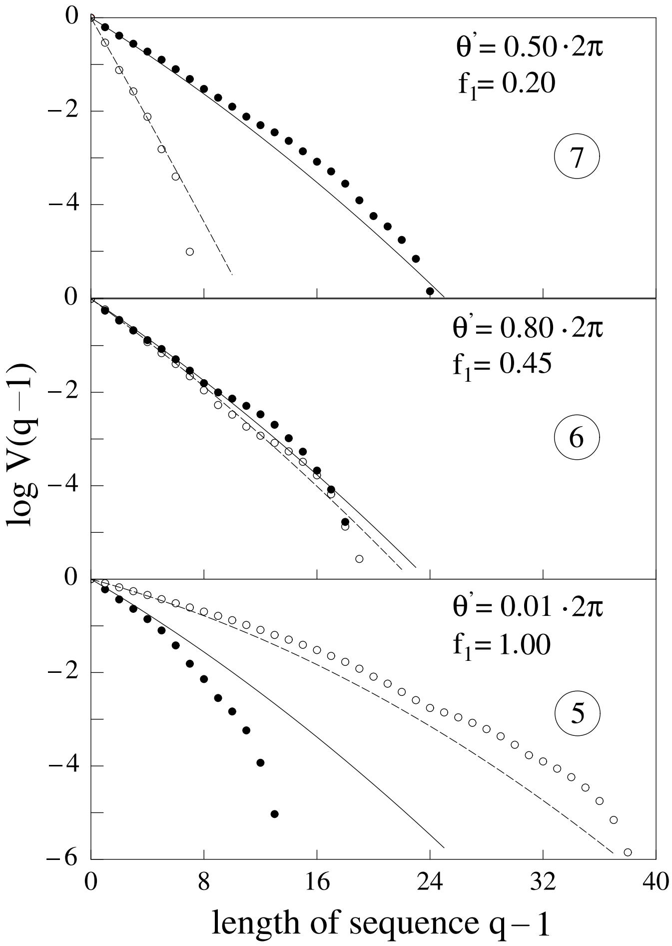

Figure 5. Probability $U(q)/U(1)$ of uninterrupted sequences of q '*on*' results (white dots) and '*off*' results (black dots). The lines show the distributions of probabilities for the ion's evolution on its drive transition, according to $V(q-1) = p_i^{q-1}$, with relaxation included. Here $i = 0$ or 1, $p_i = 1-f_i$, $B_i(1-e^{a+b}\cos\theta)$, $B_0 = (\Omega^2/2)/(\Omega^2+\Gamma\gamma)$, $B_1 = 1-B_0$, $2a = \gamma\tau = \gamma_{ph}\tau+(\Gamma/2)\tau$, $2b = \gamma\tau$, $\theta^2 = (\Omega\tau)^2 - (a-b)^2$, Ω and Γ are Rabi frequency, and decay rate of inversion, respectively, and $\gamma_{ph} = (2a-b)/\tau$ is the rate of phase diffusion of the drive light. In addition, the finite length of the trajectories of measurements has been taken into account. θ and f_1 from fit; values $f_1 < 1$ indicate redistribution, over sublevels, by cycles of spontaneous decay and re-excitation.

sublevels. The mild deviations of the observed distributions from the model curves at high q values are artefacts caused by spurious decay of the ion into its metastable $F_{7/2}$ level, combined with insufficient release from this long-living level by the light of an auxiliary diode laser.

The distributions of sequences, matching the modelled probabilities of q-times survival, prove a weakly excited superposition state of the ion ($\theta \ll \pi$) being 'set back' preferentially to its *initial* eigenstate with each probing. In terms of observed quantities, the system preferentially survives the attempts of preparation by the driving radiation.

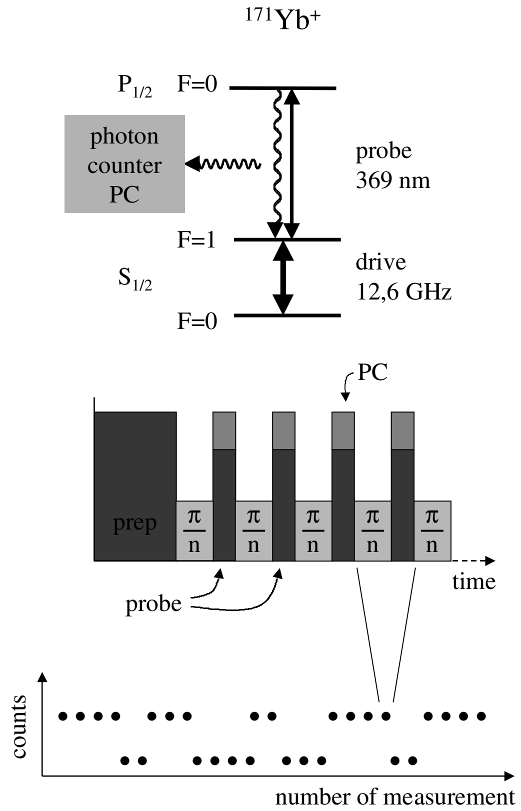

Figure 6. Alternating double resonance on individual $^{171}Yb^{+}$ ion: scheme of excitation and detection (top), temporal strategy of measurements (middle), trajectory of results (bottom).

4. Driving on a hyperfine resonance

A variant of the outlined experiment makes use of the ground-state hyperfine resonance of a single $^{171}Yb^{+}$ ion as the quantum object, in the framework of a microwave-optical double-resonance scheme [23] (figure 6). Here, relaxation does not modify the measurement, neither from spontaneous decay nor from dephasing by residual fluctuation of the driving radiation. The straightforward interpretation of the results, by equations (1) and (2), is now exact, and not an approximation, as with laser excitation of the electronic E2 resonance.

Pulses of the 12.6 GHz microwave radiation coherently and resonantly drive the ion out of its $F = 0$ ground-state level. Subsequently, the ion's state is monitored by laser pulses that alternate with the microwave pulses. If the attempted excitation into the $F = 1, m_F = 0$ state has happened, some laser light scatters off the ion on the $S_{1/2}(F = 1)$–$P_{1/2}$ resonance line and gives rise to an '*on*' signal. Failed microwave excitation is correlated with an '*off*' signal. Thus, in contrast to the previous experiment, a recorded '*off*' result correlates with the ion found in the *lower* state of its driven resonance.

The microwave pulses are set to a predetermined value of the pulse area θ. The probe-light pulses at 369 nm wavelength are generated by a Ti:sapphire laser of some 100 kHz bandwidth, downtuned from resonance by 10 MHz. Pump–probe schedule and data acquisition are controlled in real time. Occasionally pumping the ion into its $^2D_{3/2}$ level is immediately undone when the ion is repumped into its ground state by 935 nm light of a diode laser. The ion is kept

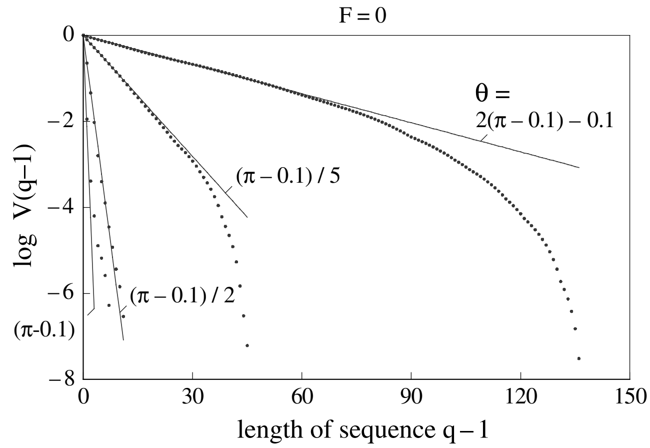

Figure 7. Probability $U(q)/U(1)$ of uninterrupted sequences of q results, *all* of them '*off*', when the ion was initially prepared in the '*off*' state ($F = 0$). The lines show the distribution of probability $V(q-1)$ for the ion *not* undergoing a flip of its spin to the 'on' state during the entire sequence. Length of trajectories: 2000 measurements of 4.9 ms driving time and 2 ms probing time (see text).

in the central electrical field node of the trap with less than 10 nm deviation, and laser-cooled below 1 mK, within the Lamb–Dicke regime (vibrational excursion $\ll$ laser wavelength). For details of this experiment, in particular for the precise setting of the microwave resonance frequency and the pulse area θ, see [23].

In each pulse pair made up of a driving and a probing pulse, again the former prepares a superposition state (except when θ is a multiple of π), and the latter subsequently reads out one of the pertaining eigenstates, $F = 0$ or 1. Series of 10^4 of such pulse pairs have yielded trajectories of results that again consist of alternating 'waiting intervals', one class of which is made up by a random number of '*on*' results, the other class by '*off*' results. As in the previous experiment, the statistical distributions of all sequences of equal results contained in these waiting intervals have been evaluated, in order to prove them compatible with the impeding effect of the measurements, as outlined in section 2. However, here they are made to fit in with the predictions of the straightforward *relaxation-free* model.

The statistical distributions of the sequences of recorded 'off' equal results are shown in figure 7. The microwave pulse area θ was subsequently set to π, $\pi/2$, $\pi/5$, and $2\pi - 0.1$. Increments $\delta\theta$ as small as 10^{-5} have turned out to be detectable; in fact, deviations from the nominal setting on the order of 3% have been observed. At high q values, the distributions no longer follow the straight lines in the logarithmic plots as predicted by the simple model, mainly owing to the finite lengths of the trajectories. Moreover, they reveal the slightly reduced probability of survival, for long '*off*' sequences, as a consequence of some spurious loss of coherence after a long accumulated interaction time.

The distributions of 'on' results, at small-to-moderate q, are also linear versus q. However, they are tainted by the non-vanishing probability of the ion to become pumped from the $F = 1$, $m_F = 0$ Zeeman sub-level into the corresponding $m_F = \pm 1$ sublevels. This spurious pumping out of the driven two-level system represents an effective loss [27], and the data must be fitted by the more complex model including relaxation [22] in order to be of evidential value.

5. Driving by effective π pulses

Another version of the microwave-optical double-resonance experiment has been carried out following an alternative strategy. Now, trains of n resonant microwave pulses of area $\theta_n = \pi/n$ are made to irradiate the ion. Since dephasing of the magnetization is negligible, each train corresponds to a π pulse of excitation ($\theta_{tot} = n\theta_n = \pi$). However, if the ion is illuminated by probe-laser pulses applied between these fractional pulses (figure 8), the evolution of the ion is supposed to be impeded and not to reach, after the nth pulse, half a revolution in configuration space ($\theta_{tot} < \pi$). Increasing the number n of fractional pulses and simultaneously decreasing their pulse area θ_0 would further reduce the total angle θ_{tot} that may be recovered from an ensemble of pulse trains: probing the ion's resonance scattering after each pulse train, recording the scattered signal '*on*' or '*off*', and averaging over the results of many trains. So far this strategy resembles the proposal by Cook [16] which has been applied in the past to that ensemble of trapped ions [17]. However, since in this experiment the effect of the probe irradiations was ignored except for the final probing, back-and-forth transitions in individual ions, or in ion pairs, were also ignored. Such a way of measurement is considered 'non-selective'. In the present experiment on a *single* ion, on the other hand, the photon-counting detector was gated open during *all* probe pulses, and the results were stored individually. In this way one may identify those trains of n results each, that represent trajectories of *uninterrupted* survival of the ion in its initial state, and discriminate those trains that include back-and-forth quantum jumps, both in one ion or in pairs of ions, that represent the ion being *retrieved*, i.e. *net* survival only.

With the number of pulse trains increasing, the normalized number of trains with uninterrupted survival of the ion approaches the probability for survival and may be compared with the analytic value,

$$P_{00}^{(s)}(n) = \cos^{2n}(\pi/2n). \tag{4}$$

This strategy is known as a 'selective' measurement, since it is equivalent to discarding all trains of n results as soon as an '*on*' result is found among the $(n-1)$ intermediate detections. Figure 9 presents, as dark grey bars, the normalized numbers of trains of n driving and probing pulse pairs that have yielded no '*on*' and only '*off*' results. In fact, the observed probability for survival *increases* with the number n of fractionating driving and interlaced probing in a pulse train, and it matches the probability of survival according to equation (4) shown in black, within the error bars. For comparison, light grey bars show the uninhibited evolution of the ion, when intermediate probing is *absent*: now, the probability of survival is supposed to vanish since the ion was irradiated, within each train, by an effective π pulse. With n not too large, null survival is also found to be in agreement with this prediction. Upon driving the ion with a highly fractionated π-pulse, the observed 10% probability of survival marks some residual dephasing of the spin moment during the extended time of interaction, $n\tau$.

In contrast, discarding the results of all $(n-1)$ intermediate measurements and registering *only* the nth probing would result in the 'non-selective' strategy of measurement. The probability of finding the recorded result of the final probing, after a pulse train, to agree with that of the probing after the previous pulse train,

$$P_{00}^{(ns)} = \tfrac{1}{2}(1 + \cos^2(\pi/n)), \tag{5}$$

is the probability of *retrieval*. It is at variance with the probability of survival and is shown as the histogram of white bars that underlie the black ones. However, the recorded data are not suitable for direct comparison: the registered trains of probing results, when evaluated in this way, inevitably include '*on*' results that are unfortunately impaired by spurious optical pumping, as was mentioned above. According to this systematic effect, their distributions

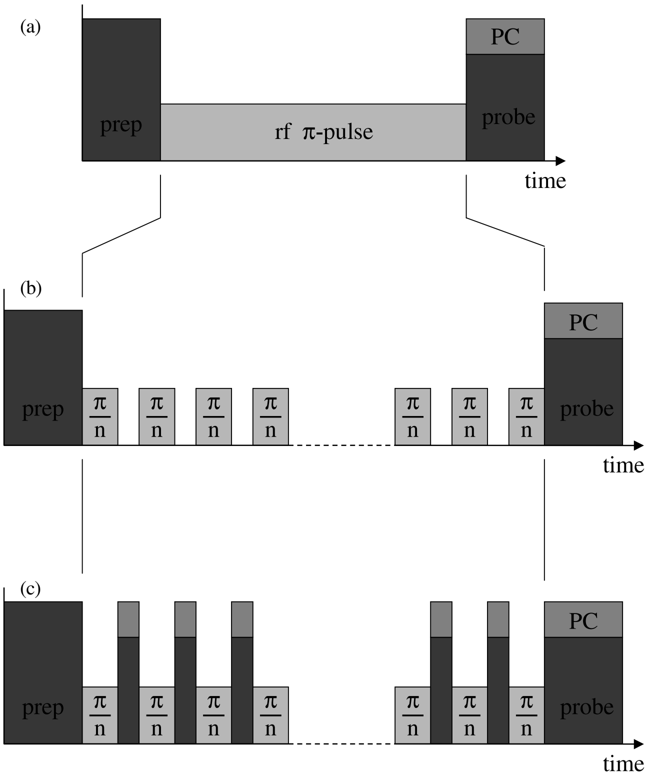

Figure 8. Scheme of measurement with (a) one driving π pulse, (b) with fractionated π pulse (n-times π/n), no intermediate probing, and (c) with π/n pulses alternating with probe pulses. PC: photon counting.

somewhat deviate from equation (5) and are not shown in the histogram. In any event, non-selective measurements are neither suited for the verification of QZE, nor of QZP [13].

For details of the experiment, in particular on the correction of systematics in the counting statistics, see [23].

6. Summary

The retardation and even the impediment of quantum evolution by reiterated identical preparation and subsequent measurement of the quantum system's state has been demonstrated on an *individual* ion's radiatively driven dipole-forbidden electronic and ground-state hyperfine resonances. Unlike with an ensemble, the evolution is here well defined, since the micro-state of the system before probing—a superposition, in general—supposed to be acquired in the course of the preparation is known, up to a phase factor. Each probing reveals an *eigenstate* in which the quantum system is found, not an expectation value. Driving by a pulse of small area takes the quantum system to a small angle of nutation in configuration space, and results in a quadratically small transition probability. Thus, the inhibition of evolution is encoded in the enhanced probability of survival in the initial eigenstate. The distribution of sequences of equal results within a long trajectory of measurements proves that the system, supposed to be prepared in a superposition state, is 'set back to an eigenstate'. Thus with a

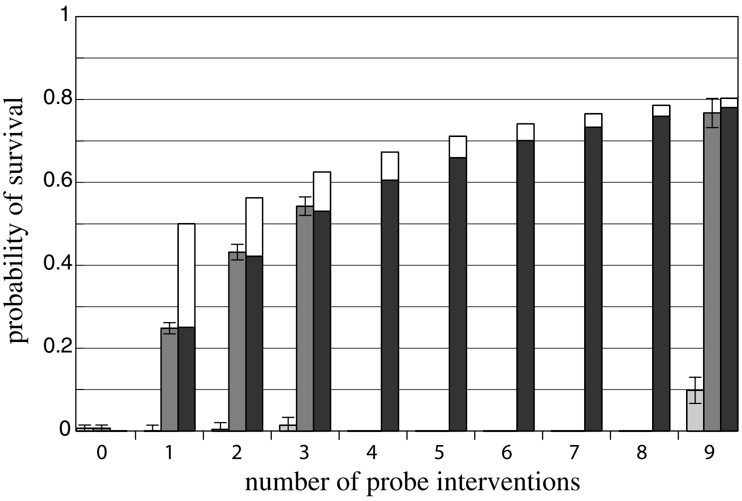

Figure 9. Probability of survival in the '*off*' state with intermediate probing (dark grey bars), and with no probing (light grey bars), versus number n of π-pulse partition. The probability is evaluated by counting as 'favourable' those measurements that show only '*off*' results in each of the n observations. Driving time 2.9 ms, probing time 3 ms. Probability of *survival* in a 'selective' measurement, calculated after equation (4) (black bars). If evaluated from the *entire* ensemble of results, including intermediate '*on*' results, such a probability of *retrieval* would represent a non-selective measurement. This probability is calculated after equation (5) (white bars).

pulse of small area applied for the driving attempts, this is most likely the eigenstate found before, i.e. the likelihood of survival increases upon driving the system by more pulses of smaller pulse area. Since measurements with '*off*' results are of the QND type, no dynamical back action is responsible for the 'set-back'. This finding means that interpretations in the spirit of the 'Heisenberg microscope' are unacceptable. In fact, the inhibition emerges from the entanglement of quantum state and scattering signal established by the almost complete correlation of the random result of quantum preparation with the subsequent near-deterministic probing.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[2] DiVincenzo D P 2000 *Fortschr. Phys.* **48** 771
Steane A M and Lucas D M 2000 *Fortschr. Phys.* **48** 839
[3] Appasamy B, Siemers I, Stalgies Y, Eschner J, Blatt R, Neuhauser W and Toschek P E 1995 *Appl. Phys.* B **60** 473
[4] von Neumann J 1932 *Mathematische Grundlagen der Quantenmechanik* (Berlin: Springer)
[5] Heisenberg W 1949 *The Principles of the Quantum Theory* (New York: Dover)
[6] Khalfin L A 1968 *Pis. Zh. Eksp. Teor. Fiz.* **8** 106 (Engl. transl. 1968 *JETP Lett.* **8** 65)
[7] Fonda, Ghirardi L, Rimini G C and Weber A 1973 *Nuovo Cimento* A **15** 689
[8] Misra B and Sudarshan E C G 1977 *J. Math. Phys.* **18** 756
[9] Beige A and Hegerfeldt G C 1996 *Phys. Rev.* A **53** 53
[10] Toschek P E and Balzer Chr 2002 *Laser Phys.* **12** 253
[11] Feynman R P, Vernon F M Jr and Hellwarth R W 1957 *J. Appl. Phys.* **28** 49
[12] Sakurai J J 1985 *Modern Quantum Mechanics* (Menlo Park, CA: Benjamin-Cummings)
[13] Home D and Whitaker M A B 1997 *Ann. Phys., NY* **258** 237
[14] Facci P and Pascazio S 2002 *Phys. Rev. Lett.* **89** 080401
[15] Maniscalco S and Messina A 2001 *Fortschr. Phys.* **49** 1027

[16] Cook R 1988 *Phys. Scr.* T **21** 49
[17] Itano W M, Heinzen D J, Bollinger J J and Wineland D J 1990 *Phys. Rev.* A **41** 2295
Itano W M, Heinzen D J, Bollinger J J and Wineland D J 1991 *Phys. Rev.* A **43** 5168
[18] Nakazato H, Namiki M, Pascazio S and Rauch H 1996 *Phys. Lett.* A **217** 203
[19] Spiller T P 1994 *Phys. Lett.* A **192** 163
[20] Alter O and Yamamoto Y 1997 *Phys. Rev.* A **55** 2499
[21] Braginsky V B and Khalili F Ya 1992 *Quantum Measurement* (Cambridge: Cambridge University Press)
[22] Balzer Chr, Huesmann R, Neuhauser W and Toschek P E 2000 *Opt. Commun.* **180** 115
[23] Balzer Chr, Hannemann Th, Reiß D, Wunderlich Chr, Neuhauser W and Toschek P E 2002 *Opt. Commun.* **211** 235
[24] See, e.g.
Fano U and Fano L 1970 *Physics of Atoms and Molecules* (Chicago, IL: University of Chicago Press)
[25] Fawcett B C and Wilson M 1991 *At. Nucl. Data Tables* **47** 241
[26] Torrey H C 1949 *Phys. Rev.* **76** 1059
[27] Balzer Chr, Hannemann Th, Reiss D, Neuhauser W, Toschek P E and Wunderlich Ch 2002 *Laser Physics at the Limits* (Berlin: Springer) p 233