Ion-Trap Quantum Logic Using Long-Wavelength Radiation

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A quantum information processor is proposed that combines experimental techniques and technology successfully demonstrated either in nuclear magnetic resonance experiments or with trapped ions. An additional inhomogenenous magnetic field applied to an ion trap (i) shifts individual ionic resonances (qubits), making them distinguishable by frequency, and (ii) mediates the coupling between internal and external degrees of freedom of trapped ions. This scheme permits one to individually address and coherently manipulate ions confined in an electrodynamic trap using radiation in the radiofrequency or microwave regime.

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Quantum information processing (QIP) holds the promise of extending today's computing capabilities to problems that, with increasing complexity, require exponentially growing resources in time and/or the number of physical elements [1]. The computation of properties of quantum systems themselves is particularly suited to be performed on a quantum computer, even on a device where logic operations can be carried out only with limited precision [2]. Elements of quantum logic operations have been successfully demonstrated in experiments using ion traps [3–5], cavity quantum electrodynamics [6], and in the case of nuclear magnetic resonance (NMR) even algorithms have been performed [7]. Whereas quantum computation with nuclear spins in macroscopic ensembles can most likely not be extended beyond about ten qubits (quantum mechanical two-state systems) [8], ion traps do not suffer from limited scalability *in principle* and represent a promising system to explore QIP experimentally. They can be employed to also investigate fundamental questions of quantum physics, for example, related to decoherence [9] or multiparticle entanglement [3]. However, they still pose considerable experimental challenges.

Two internal states of an *individual* ion are used as a qubit. The vibrational motion of a *collection* of trapped ions serves as the "bus-qubit" and permits conditional dynamics between individual qubits [10]. In order to couple internal and motional degrees of freedom of a trapped atom, the atom has to experience an appreciable variation of the field that drives the internal transition over the extent of its spatial wave function. A measure for the strength of the field gradient relative to the atoms spatial extend $\Delta z \equiv \sqrt{\hbar/(2m\omega_l)}$ is the Lamb-Dicke parameter (LDP) $\eta = \Delta z 2\pi/\lambda$. ($\lambda$ is the wavelength of the applied radiation; the atom with mass m is trapped in a harmonic potential characterized by angular frequency ω_l.) For typical qubit transitions and useful trap frequencies, this parameter has an appreciable nonzero value only for driving radiation in the optical domain.

Consequently, all schemes for ion trap QIP used (for example, [3–5]) and suggested [10,11] have in common that laser light is necessary to drive qubit transitions. Involved optical setups are required to cool the vibrational motion of the ions, and to prepare, coherently manipulate, and read out the qubit states. It is desirable to find simpler methods for the manipulation of well isolated qubits in ion traps, methods that require a smaller number of laser beams and sources, and are less demanding regarding the specifications of beam quality and pointing stability, and frequency and intensity stability.

Another important issue when trying to implement a quantum information processor and prerequisite for further studies using several ions is the addressing of individual qubits out of a large collection of ions. In order to perform operations on an optically driven transition between qubit states of an individual ion, strongly focused laser light must be aimed at only the desired ion [12]. Different approaches have been used and proposed instead [13] to circumvent practical and fundamental difficulties arising from such an addressing scheme.

Techniques for generating radiation with long coherence time that are experimentally challenging and/or require intricate setups in the optical domain are well established in the radiofrequency (rf) or microwave (mw) domain where commercial off-the-shelf components can be used. Technological resources developed over decades in this frequency range have been used in an inventive way for NMR methods and contributed to the impressive and fast success of NMR in QIP. It would be desirable to use these resources for ion trap QIP also. Two obstacles have precluded rf or mw radiation from being used for the manipulation of qubits in ion traps (for example, comprised of two hyperfine states): (i) The LDP is essentially zero for mw radiation and useful trap frequencies. Thus, coupling of internal and external degrees of freedom is not possible. (ii) mw radiation cannot be focused such that individual ions can be addressed.

Here, we show that an additional magnetic field gradient applied to an electrodynamic trap (i) introduces a coupling between internal and motional states even for rf or mw radiation and (ii) serves to individually shift ionic

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qubit resonances thus making them distinguishable in frequency space. With the introduction of this field, all optical schemes devised for QIP in ion traps can be applied in the rf or mw regimes also.

We consider ionized atoms confined in the initially field-free region along the symmetry axis of an ac-quadrupole field of a linear electrodynamic trap [14]. The ions are trapped due to a pseudopotential that is harmonic in the center of the trap and described by $V = \frac{1}{2}m\omega_r^2(x^2 + y^2) + \frac{1}{2}m\omega_z^2 z^2$, where the angular frequencies ω_r and ω_z characterize the trapping potential in the radial and the axial direction, respectively. If more than one ion is trapped, the equilibrium positions are determined by the condition that trapping force and Coulomb forces add to zero for each ion. As long as $\omega_r/\omega_z \gtrsim 0.73N^{0.86}$, where N is the number of ions in the trap, the x and y components of the equilibrium positions vanish [11] and the ions form a linear chain characterized by axial vibrational eigenfrequencies ω_l ($l = 1, \ldots, N$). Such a linear configuration will be considered in what follows.

Applying a magnetic field $\vec{B}_{\rm dc}$ to the trap leads to Zeeman energies $\varepsilon_0(B_{\rm dc})$ and $\varepsilon_1(B_{\rm dc})$ of the internal qubit states $|0\rangle$ and $|1\rangle$. $\vec{B}_{\rm dc} = (bz + b_0)\hat{z}$ is chosen with magnetic field gradient $b \equiv \frac{\partial B_{\rm dc}(z)}{\partial z} \neq 0$ and constant offset b_0, leading to an individual, position dependent Zeeman shift for each ion such that the qubit resonance frequency $\omega(z) = \{\varepsilon_1[B_{dc}(z)] - \varepsilon_0[B_{dc}(z)]\}\hbar^{-1}$. With $\partial_z\varepsilon_1 \neq \partial_z\varepsilon_0$, this Zeeman shift gives rise to a state dependent force in an inhomogeneous magnetic field. Thus internal state transitions cause a slight displacement of the ion, and internal and motional degrees of freedom are coupled. Since the spatial excursion of an ion is of the order $\Delta z\sqrt{2\bar{n} + 1}$ ($\bar{n}$ is the mean vibrational quantum number at the Doppler limit), this additional Zeeman potential is linear in the ion's displacement to very good approximation.

As in the proposal by Cirac and Zoller [10], a collective vibrational mode is employed as a means of communication (bus-qubit) between otherwise isolated internal qubit states of the ions. The Hamiltonian $H = \frac{1}{2}\hbar\omega(z)\sigma_z + \hbar\omega_l a_l^\dagger a_l$ describes the qubit states of a particular ion coupled to vibrational mode l [$\sigma_z = |1\rangle\langle 1| - |0\rangle\langle 0|$.] When expanded to first order in the axial position operator, $\zeta\Delta z(a_l^\dagger + a_l)$, it reads

$$H = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega_l a_l^\dagger a_l + \frac{1}{2}\hbar\omega_l\varepsilon_c(a_l^\dagger + a_l)\sigma_z\,, \tag{1}$$

with $\varepsilon_c \equiv \zeta(\Delta z|\partial_z\varepsilon_1 - \partial_z\varepsilon_0|)/(\hbar\omega_l)$. Here, ζ is the expansion coefficient of the displacement of the ion to be addressed in terms of the normal mode coordinate. For the center-of-mass mode $\zeta = 1/\sqrt{N}$ and for any other mode $\zeta \approx 1/\sqrt{N}$. The qubit's resonance frequency at its equlibrium position is denoted by ω_0. It is useful to perform the unitary transformation $\tilde{H} = e^S He^{-S}$ with $S = \frac{1}{2}\varepsilon_c(a_l^\dagger - a_l)\sigma_z$ and, after dropping constant terms, we obtain $\tilde{H} = \frac{1}{2}\hbar\omega_0\sigma_z + \hbar\omega_l a_l^\dagger a_l$, i.e., in the transformed Hamiltonian, the coupling between internal degree of freedom and vibrational mode has been eliminated. The transformed operators are given by $\tilde{a}_l = a_l - \frac{1}{2}\varepsilon_c\sigma_z$, $\tilde{a}_l^\dagger = a_l^\dagger - \frac{1}{2}\varepsilon_c\sigma_z$, $\tilde{\sigma}_+ = \sigma_+ e^{(a_l^\dagger - a_l)}$, and $\tilde{\sigma}_- = \sigma_- e^{-\varepsilon_c(a_l^\dagger - a_l)}$.

When an ion interacts with an additional electromagnetic field of frequency ω_M, this leads to an interaction term,

$$H_M = \frac{1}{2}\hbar\Omega_R(\sigma_+ + \sigma_-)\,(e^{i[\eta(a_l + a_l^\dagger) - \omega_M t]} + \text{H.c.})\,, \tag{2}$$

where $\eta = \zeta\sqrt{(\hbar k_z^2/2m\omega_l)}$ is the LDP for N ions and $\Omega_R = \frac{\vec{\mu}\cdot\vec{B}_M}{\hbar}$ is the Rabi frequency (here of a magnetic dipole transition) characterizing the coupling strength. The magnetic dipole moment operator is denoted by $\vec{\mu}$, the magnitude of the wave vector in axial direction $k_z = \frac{\omega_M}{c}\cos\theta$, where θ is the angle between the incident beam and the trap axis, and B_M is the magnetic amplitude of the electromagnetic field.

The transformed interaction $\tilde{H}_M = e^S H_M e^{-S}$ is given by

$$\begin{aligned}\tilde{H}_M = {}& \frac{1}{2}\hbar\Omega_R(\sigma_+ e^{\varepsilon_c(a_l^\dagger - a_l)} \\ & + \sigma_- e^{-\varepsilon_c(a_l^\dagger - a_l)}) \\ & \times (e^{i[\eta(a_l + a_l^\dagger - \varepsilon_c\sigma_z) - \omega_M t]} + e^{-i[\eta(a_l + a_l^\dagger - \varepsilon_c\sigma_z) - \omega_M t]})\,.\end{aligned} \tag{3}$$

It is useful to perform a further transformation to the interaction picture with respect to $\tilde{H}$. With detuning $\Delta = \omega_M - \omega_0$, this leads to

$$\begin{aligned}\tilde{H}_M = {}& \frac{1}{2}\hbar\Omega_R(\sigma_+ e^{-i(\Delta t + 2\eta\varepsilon_c)} \\ & \times e^{i[(\eta + i\varepsilon_c)a_l + (\eta - i\varepsilon_c)a_l^\dagger]} + \text{H.c.})\,,\end{aligned} \tag{4}$$

where terms oscillating with frequency $\pm(\omega_M + \omega_0)$ have been dropped (rotating wave approximation). For $\varepsilon_c > 0$, i.e., when a magnetic field gradient is applied, the LDP η can be replaced by a complex one, $\eta + i\varepsilon_c$. This complex parameter can be decomposed into its absolute value $\eta' = \sqrt{\eta^2 + \varepsilon_c^2}$ and its phase, that in turn can be accounted for by incorporating it into the arbitrary initial conditions of the phonon operator's time dependence. Because σ_+ also is defined only up to an arbitrary phase, the phase factor $e^{-2i\eta\varepsilon_c}$ can be appended to this operator, and what remains is the usual field-ion interaction governed by an *effective* LDP η'.

When mw radiation is used to drive internal transitions of a qubit in a *usual* ion trap (i.e., without magnetic field gradient), then the LDP η is very small ($\eta \approx 7 \times 10^{-7}$ for 40 Yb^+ ions with transition frequency $\omega_0 = 2\pi$ 12.6 GHz at a trap frequency of $2\pi \times 100$ kHz). Thus, coupling internal and external degrees of freedom of an ion is not possible with mw radiation in the usual scheme. However, it *is* possible with an additional magnetic field gradient: Even when $\eta \approx 0$, then still $\eta' \approx \varepsilon_c > 0$. All operations (including, for example, sideband cooling) that require coupling between internal states and vibration of the ion string, usually carried out with optical fields, can now be implemented using microwave radiation. In Table I some values of ε_c are listed. The required values for $|\partial_z \varepsilon_1 - \partial_z \varepsilon_0|$ will be considered in what follows.

In addition to coupling internal and external degrees of freedom of the ions, the field gradient applied to the ion trap serves to distinguish qubits by separating their resonance frequencies. The magnitude of the magnetic field gradient determines the frequency separation of qubit resonances in adjacent ions: The resonance frequency of a particular qubit is shifted relative to a neighboring ion by $\delta\omega = |\kappa_1(B_{dc}) - \kappa_0(B_{dc})| \frac{\mu_B}{\hbar} b \delta z$, where the distance between two ions is given by [15] $\delta z \approx z_0 \frac{2}{N^{0.559}}$ and $z_0 = [e^2/(4\pi\varepsilon_0 m \omega_z^2)]^{1/3}$, and the coupling constants κ_1 and κ_0 that characterize the particular hyperfine states chosen for the qubit can be obtained from the Breit-Rabi formula. To be concrete, we consider the $F = 1$, $m_F = +1$, and $F = 0$ hyperfine states [5] of $^{171}Yb^+$ in what follows. In the weak field limit, $\frac{\mu_B B_{dc}}{E_{hfs}} \ll 1$, the Breit-Rabi formula gives $\kappa_1 = 1$ and $\kappa_0 = 0$, respectively, whereas for $\frac{\mu_B B_{dc}}{E_{hfs}} = 1$ we obtain $\kappa_1 = 1$ and $\kappa_0 = -0.89$ due to the nonlinear Zeeman effect.

By choosing the magnetic field gradient b appropriately, the ions' qubit resonances can be well separated, and any chosen ion can be addressed by switching the frequency of the driving mw field. If, in the usual addressing scheme (using focused laser beams), it were possible to exclusively illuminate a single ion, that is, if *resonant* unwanted excitation could be avoided completely, then the remaining source of unwanted excitation would be *nonresonant* excitation of neighboring resonances (motional sidebands or carrier) of the ion being addressed. Our numerical studies show that only the resonances next to the driven one contribute appreciably to errors introduced by nonresonant excitation [16]. In the scheme proposed here, unwanted *resonant* excitation does not occur. We require the frequency separation between the sideband resonance corresponding to the highest axial vibrational frequency ω_N [17] of an arbitrary ion and the sideband resonance corresponding to ω_l (the bus-qubit) of its neighboring ion to be larger than ω_l. This corresponds to the frequency separation of resonances in the usual scheme, and the probability for spurious excitation of neighboring ions in the linear chain is equal to or smaller than the probability of unwanted excitation of a resonance close to the desired one. Given this requirement, the new scheme does not impose a new upper limit on the fidelity of basic quantum logic operations due to an unwanted excitation, and an estimate for the necessary B-field gradient is obtained from $b \geq \frac{\hbar}{2\mu_B} \frac{1}{|\kappa_1 - \kappa_0|} (\frac{4\pi\varepsilon_0 m}{e^2})^{1/3} \omega_z^{5/3} (4.7N^{0.56} + 0.5N^{1.56})$. Here, $\omega_l = \omega_z$ and the highest vibrational frequency, ω_N, has been approximated by the empirical law $\omega_N = (2.7 + 0.5N)\omega_z$ valid for $5 \leq N \leq 100$ that was deduced from numerical calculations of ω_N with N ranging from 2 to 100 [16].

In Table I, values of the field gradient necessary to spectrally separate qubit resonances of $^{171}Yb^+$ are listed for different trap frequencies and numbers of ions in one trap, respectively. Magnetic field gradients of the magnitude required to separate the ion resonances are well within capabilities of current technology. Reichel, Hänsel, and Hänsch [18], for example, achieved gradients of about $300 \frac{T}{m}$ over a distance of 50 μm (which corresponds roughly to the axial extension of a string of 40 $^{171}Yb^+$ ions at a trap frequency of $2\pi \times 500$ kHz) using microfabricated conductors. Gradients up to 8000 T/m are realistic in the near future [19].

The field gradient necessary to separate the ion resonances grows with the number of ions stored in the trap. This will limit the number of qubits available in a single trap [20]. However, the scalability of a possible future ion trap quantum computer does not rely on the storage of all qubits in a single trap. Instead, arrays of traps communicating via "flying" qubits (photons) [21] have been envisaged. Communication between different traps can be established by the use of photons that transfer quantum information, e.g., via optical fibers.

We have investigated in detailed numerical calculations possible detrimental effects associated with a magnetic field gradient applied to a linear ion trap [16]. The dependence of the equilibrium position of each ion on magnetic forces that in turn depend on its internal state leads to a change of vibrational and internal transition

TABLE I. The magnetic field gradient $b(b_0 = 0)$ needed to separate the resonances of $^{171}Yb^+$ ions, the coupling constant ϵ_c (analogous to the Lamb-Dicke parameter), and the average error $1 - f$ for an arbitrary qubit rotation (for Rabi frequency $\Omega_R = \frac{1}{10}\omega_z$) for different trap frequencies and numbers of ions. Gradients up to 8000 T/m are within reach of current experiments [18,19].

	$N = 10$			$N = 20$			$N = 40$		
	b (T/m)	ϵ_c	$1 - f$	b (T/m)	ϵ_c	$1 - f$	b (T/m)	ϵ_c	$1 - f$
$\omega_z/2\pi = 100$ kHz	9.89	0.0075	3.4×10^{-6}	22.1	0.012	5.2×10^{-5}	54.7	0.021	1.1×10^{-3}
$\omega_z/2\pi = 1$ kHz	459	0.011	1.6×10^{-5}	1030	0.018	2.4×10^{-4}	2540	0.031	4.9×10^{-3}

frequencies when any one of the qubit internal states is changed. As a consequence, the transition frequency $\omega_0^k(\{a_j\})$, $j \in \{1,\ldots,N\}$, $j \neq k$ of a given ion k depends slightly on the internal states labeled a_j of other ions. We calculated the mean transition frequency $\bar{\omega}_0^k = \frac{1}{2^{N-1}} \sum_{a_j, j\neq k} \omega_0^k(a_1, a_2, \ldots)$ taking into account the order of N^2 randomly chosen internal state configurations. The spread of ω_0 around its mean value $\bar{\omega}_0$ is well characterized by a normal distribution with standard deviation σ_k but which is cut off at some value with typical size $2\sigma_k$ (maximum deviation from the mean value). The distribution of ω_0 can be regarded as the width of the qubit transition. The uncertainty in resonance frequency will only negligibly affect coherent manipulation of internal qubits and bus-qubit as long as this uncertainty is much smaller than the Rabi frequency Ω_R between qubit states: A measure for the reliability of a quantum gate is the error $1 - f$, with average fidelity $f = \frac{\Omega_R}{2\pi^2} \int_0^1 d\alpha \int_0^{2\pi} d\varphi \int_0^{\pi/\Omega_R} dt |\langle\Psi_f|\Psi_r\rangle|^2$, where $|\Psi_r\rangle$ is the state obtained after an imperfect one-qubit rotation, and $|\Psi_f\rangle$ denotes the final state that would be obtained if this operation were perfect. Averaging over initial states $|\Psi_i\rangle = \alpha|0\rangle + e^{i\varphi}\sqrt{1-\alpha^2}|1\rangle$ and pulse duration $1 - f = \frac{41}{120}(\sigma^2/\Omega_R^2)$ is obtained ($\sigma = 1/N \sum_{k=1}^{k=N} \sigma_k$). The values of $1 - f$ for $\Omega_R = \frac{1}{10}\omega_z$ listed in Table I show that the effect of the frequency change on the fidelity of quantum logic operations is well below technological limits of current ion trap setups (for example, [3,12]).

All schemes devised for coherent manipulation of qubits in usual traps can still be applied here. In particular, fast quantum gates as suggested by Jonathan, Plenio, and Knight [22] can be performed (the condition in our notation is $\Omega_R = \omega_l$). Sideband cooling to the vibrational ground state can be implemented in the usual way, except that now microwave radiation is used to drive the so-called red sideband of the hyperfine transition. When, for example, Yb^+ is used, two commercial light sources in conjunction with microwave radiation [16] are sufficient for Doppler and sideband cooling of the bus-qubit, state preparation, coherent manipulation, and detection of qubits.

In conclusion, the scheme proposed here permits coherent manipulation and individual addressing of trapped ions using microwave radiation and can be implemented using current ion-trap technology in conjunction with techniques from NMR spectroscopy. Even multi-qubit operations should be possible using the present scheme.

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