Quantum gates in rare-earth-ion doped crystals

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Outline

- Rare-earth-ion doped crystals as quantum computer hardware
- Experimental results
- Current status and outlook

Requirements for quantum computing

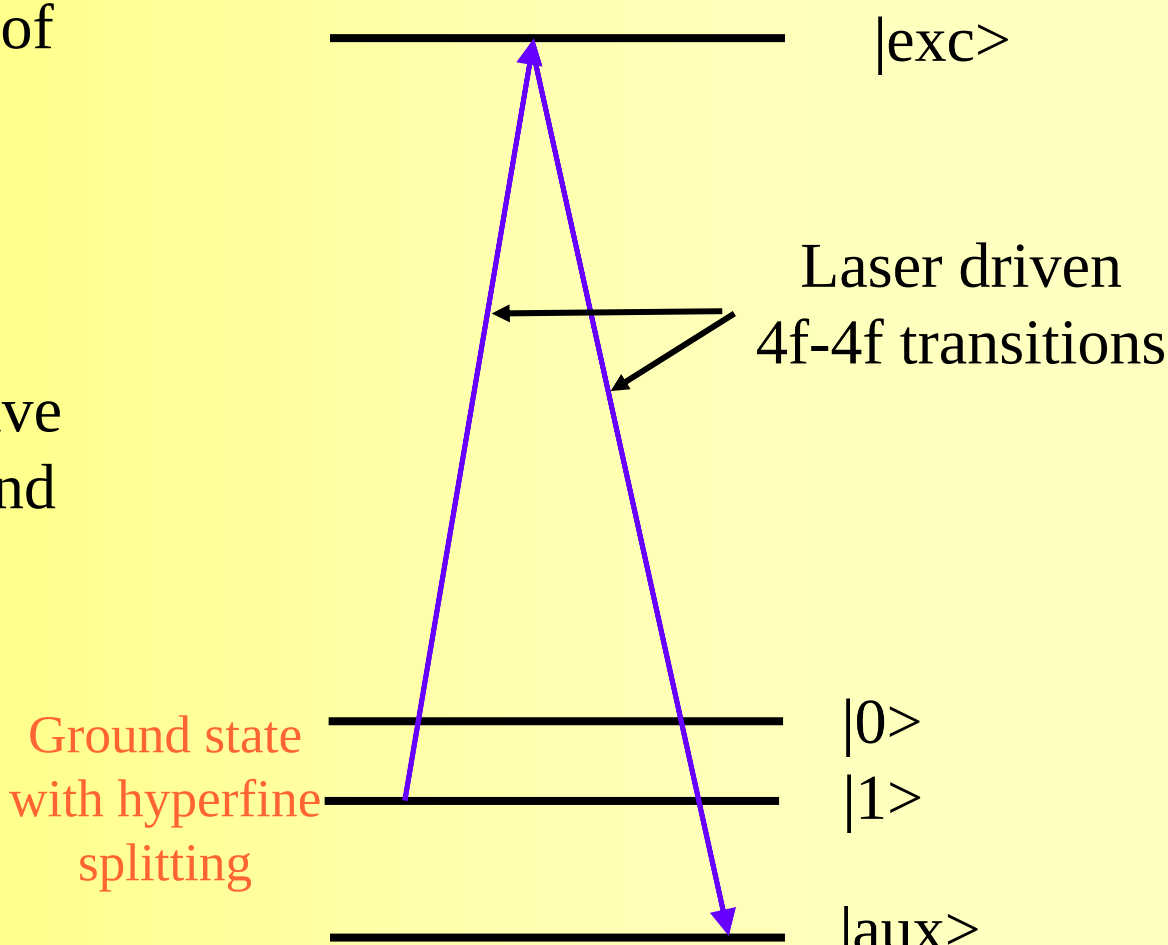
- **Coherent two-level systems acting as qubits**
- Possibility to manipulate the qubits individually (single qubit operations)
- Coupling between any two qubits (two-bit gates)
- Possibility for reliable read-out of the individual qubits
- Scalability

The rare-earth-ions hyperfine states are used as qubit states

- Long coherence times of the optical transitions (100 μs to 6 ms)

- At 4 Kelvin the ground state hyperfine levels have ms-s coherence times and very long lifetimes (~ hours)

- The duration of a laser π-pulse, ~ 1 μs



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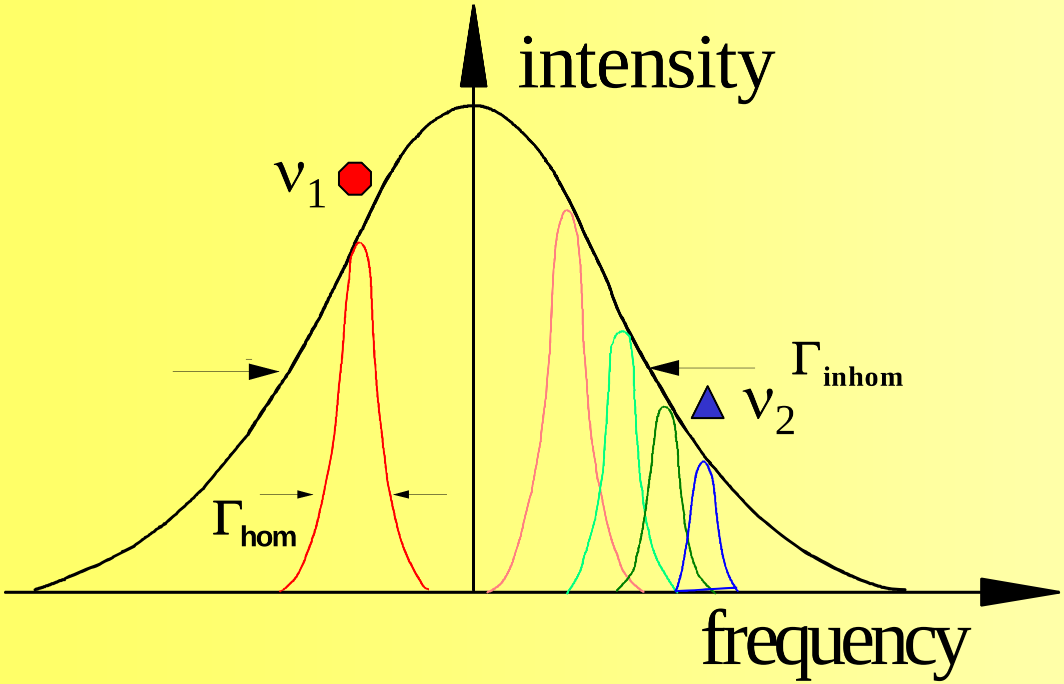
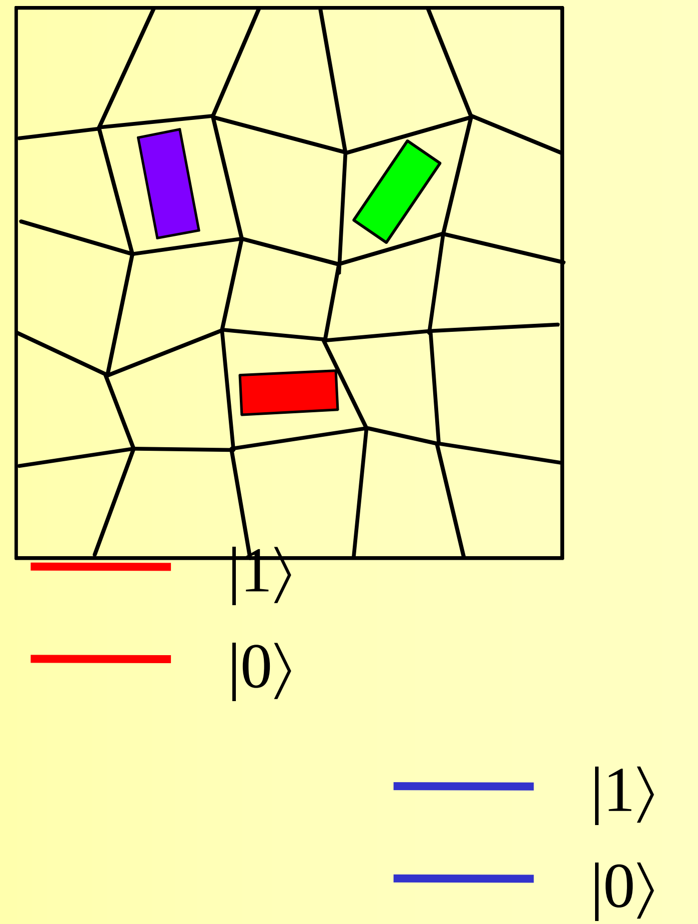
4f-4f absorption line from dopant ions in a rare earth doped inorganic crystal

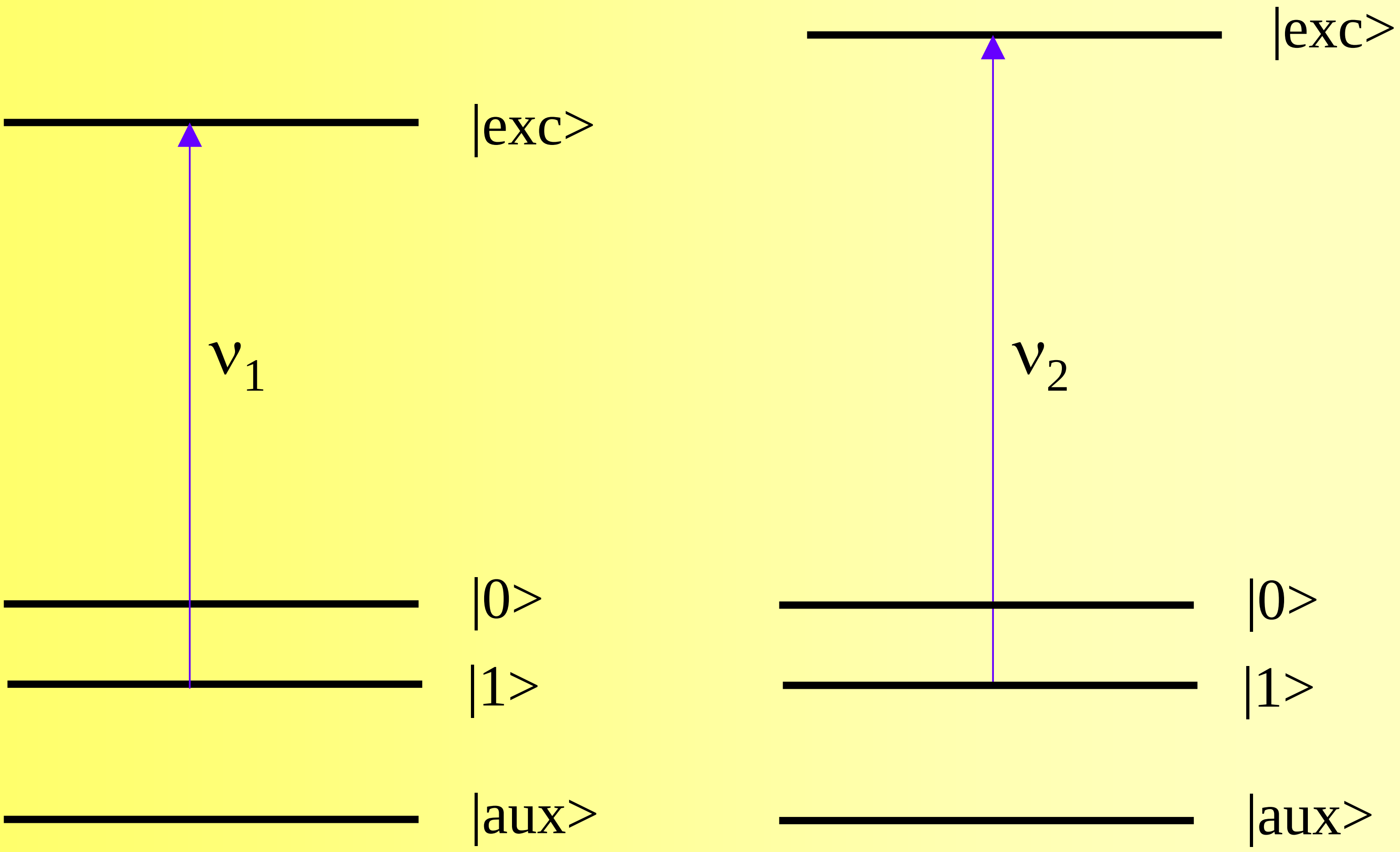


Conceptual picture of crystal with dopant ions



- Narrow homogeneous line-widths (1-10 kHz)
- Large inhomogeneous line-widths (1-200 GHz)

Addressing two different qubits in a rare-earth quantum computer



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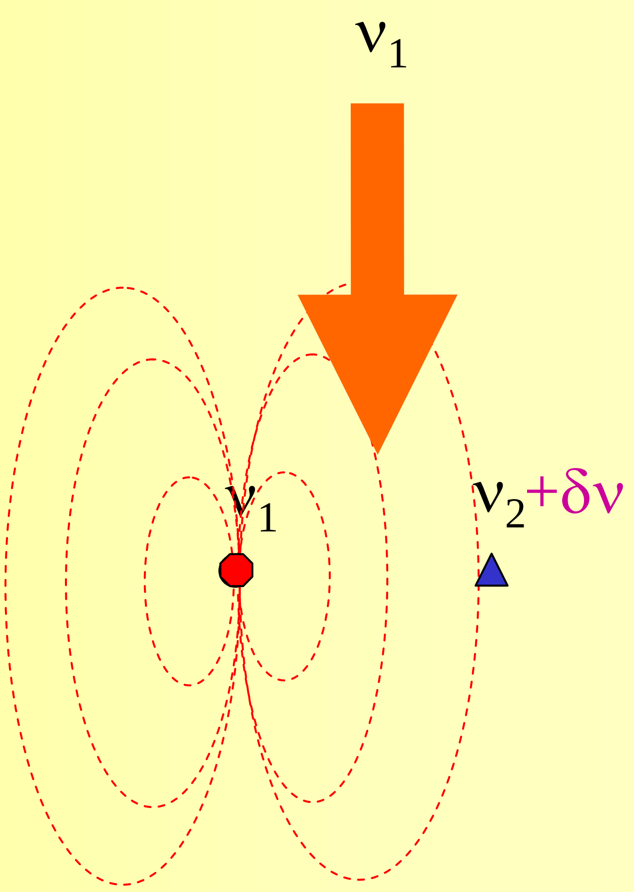
Dipole-dipole interaction

1. Two ions absorbing at different frequencies are located close to each other in the crystal lattice. In a non-centrosymmetric site the ions will have a permanent electric dipole moment and ground and excited state dipole moments can be different



2. One of the ions is excited on its optical transition

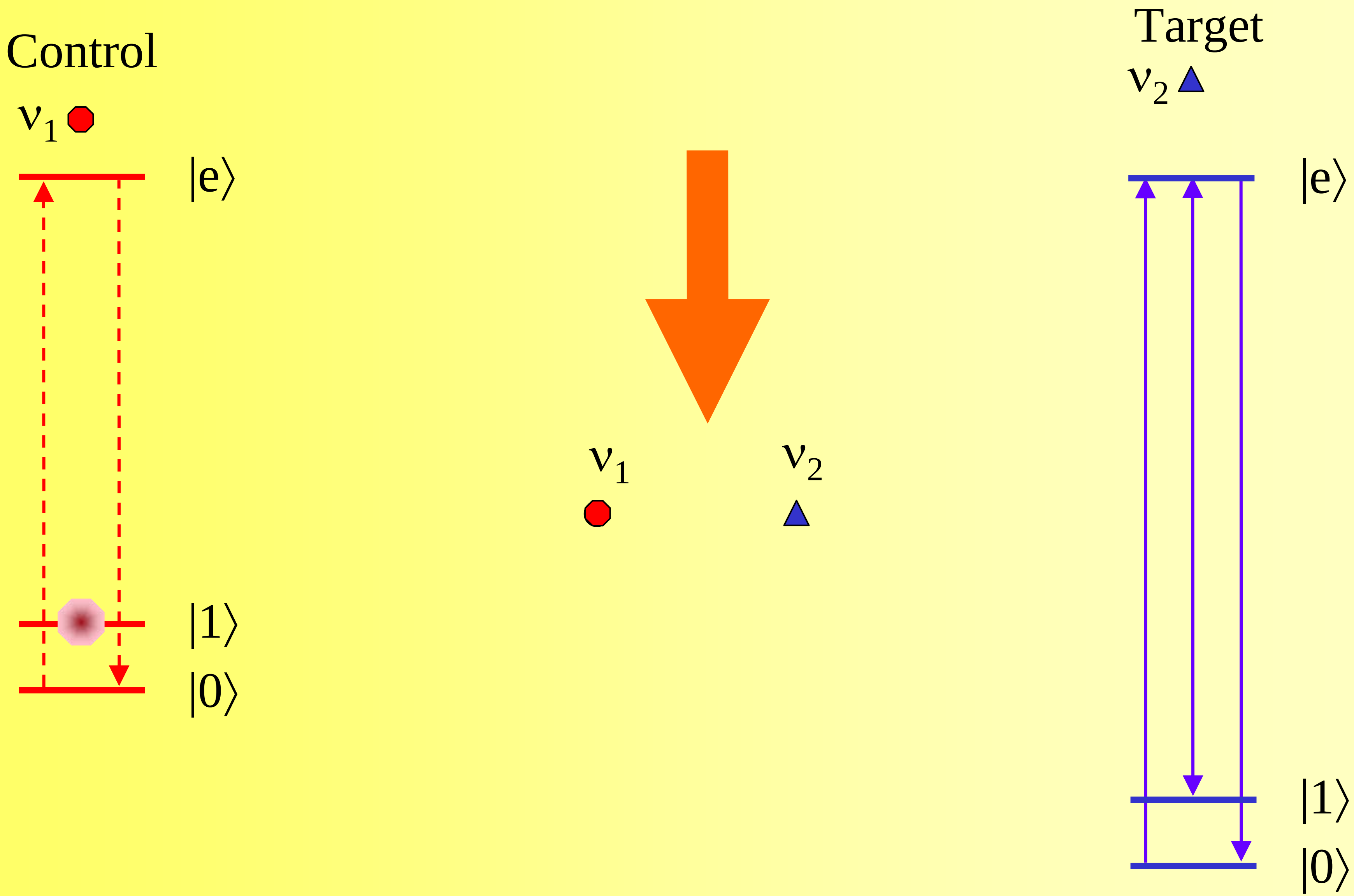
3. This change in dipole moment is sensed by the other ion causing its absorption frequency to change.

Dipole-dipole interaction strength in rare-earth crystals

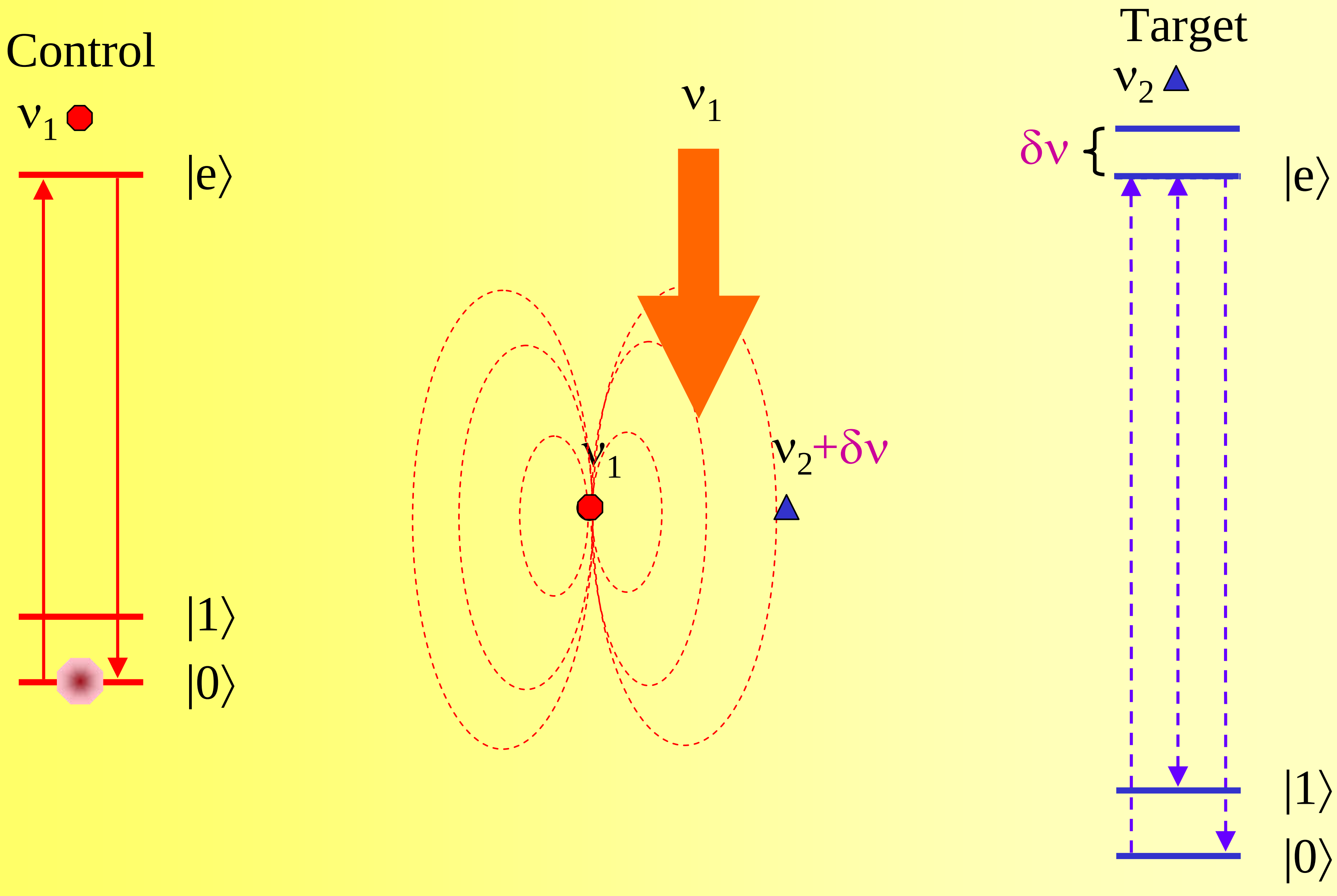
- Approximate numbers

Ion distance	frequency shift
100 nm	1 line width
10 nm	1000 line widths
1 nm	1000000 line widths

Controlled-NOT quantum gate



Controlled-NOT quantum gate



Requirements for quantum computing

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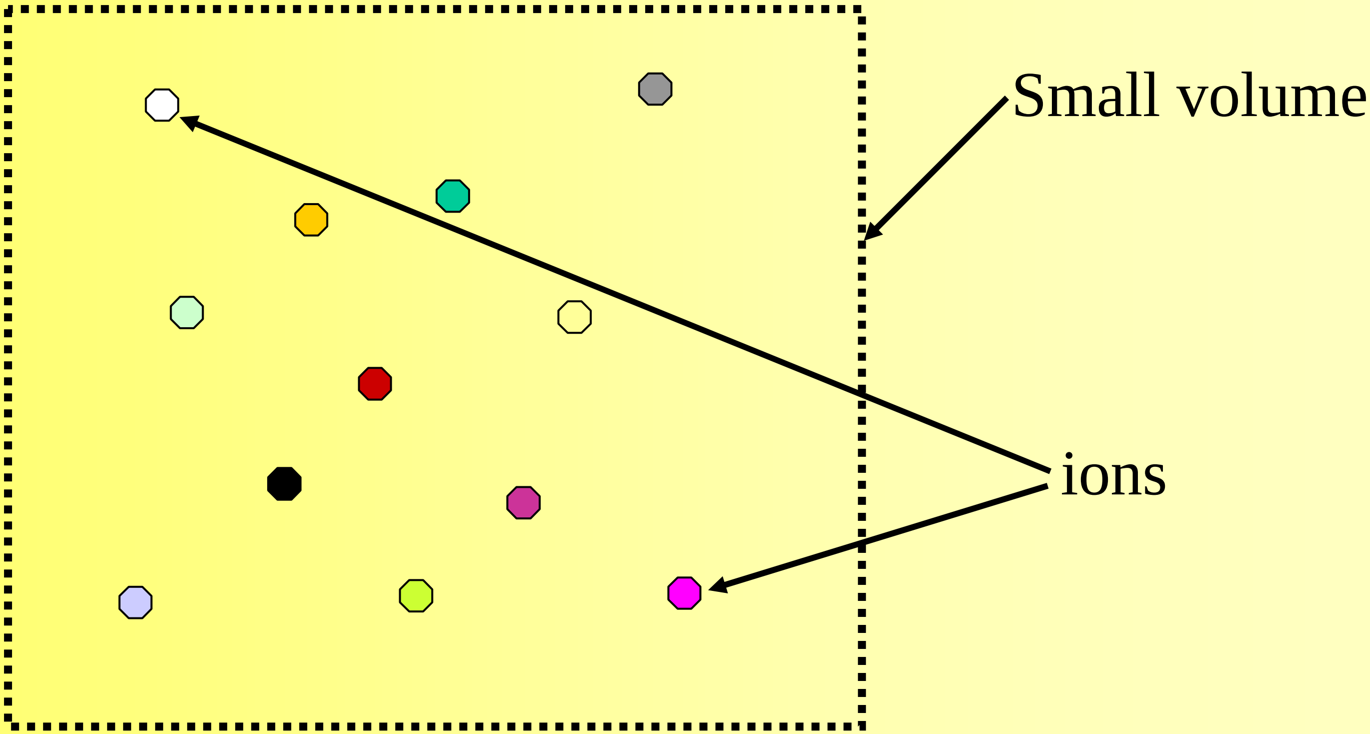
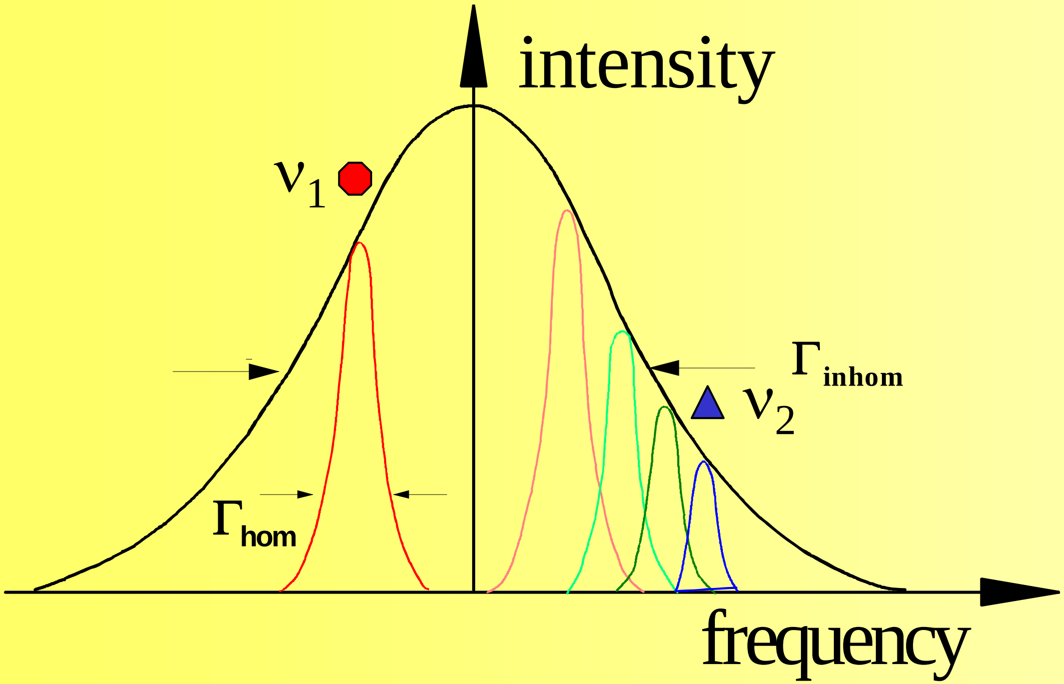
All ions in a randomly selected small volume will interact strongly



All ions interact strongly, but detecting single ions is difficult

Absorption line from dopant ions in a rare earth doped inorganic crystal



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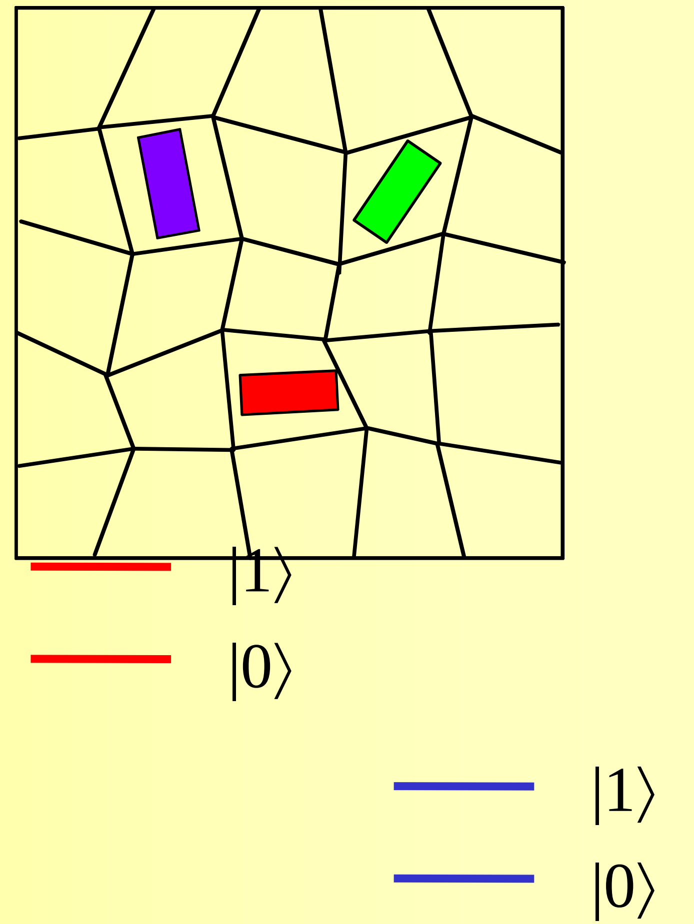

- Narrow homogeneous line-widths (1-10 kHz)
- Large inhomogeneous line-widths (1-200 GHz)

Challenges with multiple instance approaches

- All ions in a qubit (i.e. all QC instances) must have identical wave functions
- Both optical and hyperfine transitions are inhomogeneously broadened, ions in different instances may experience different laser field strengths and couple differently to the field
- Ion-ion interaction may differ between instances
- It may be possible to construct pulses which compensate for this but this adds to the operation time and increase decoherence

Requirements for quantum computing

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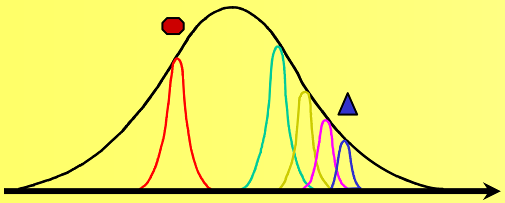
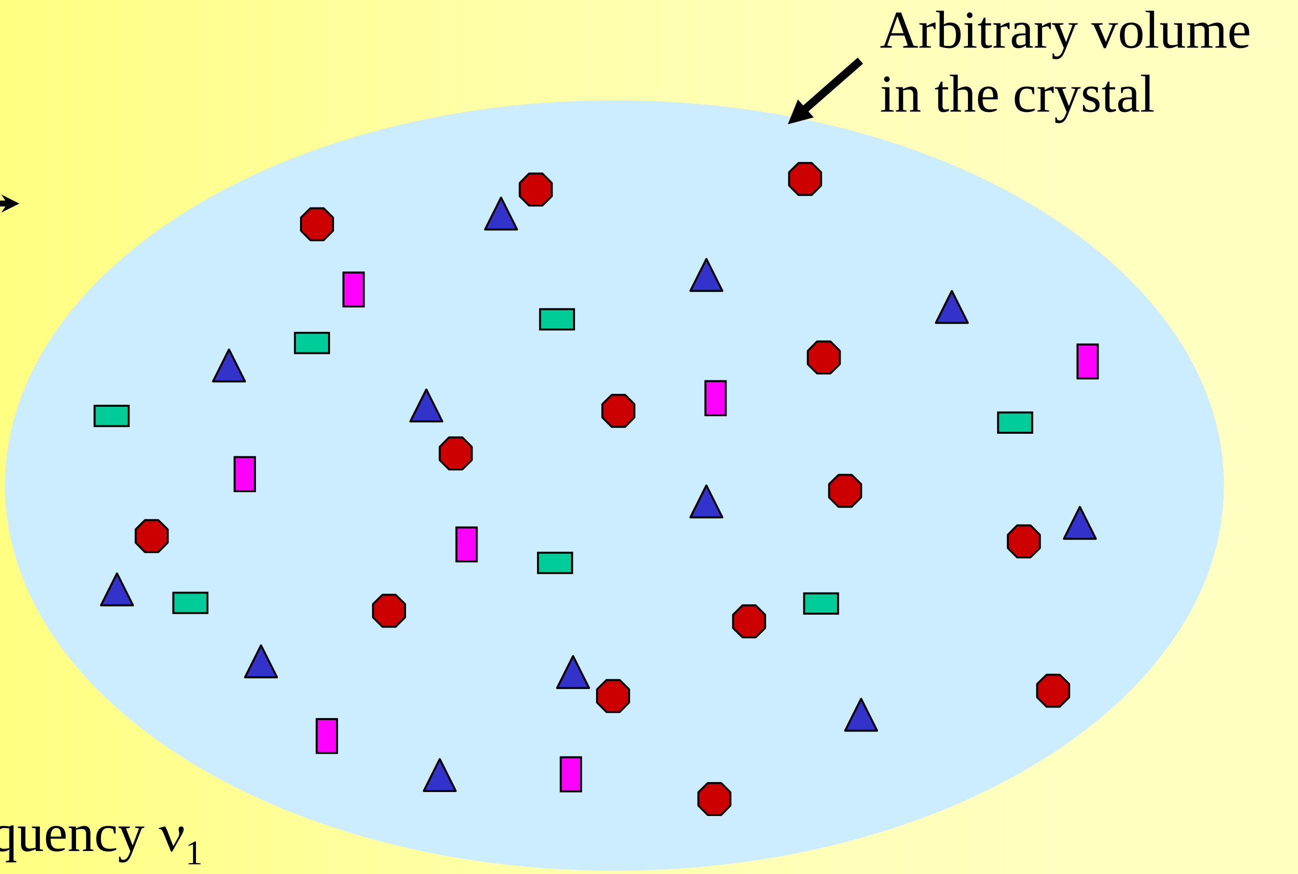
Qubit distillation

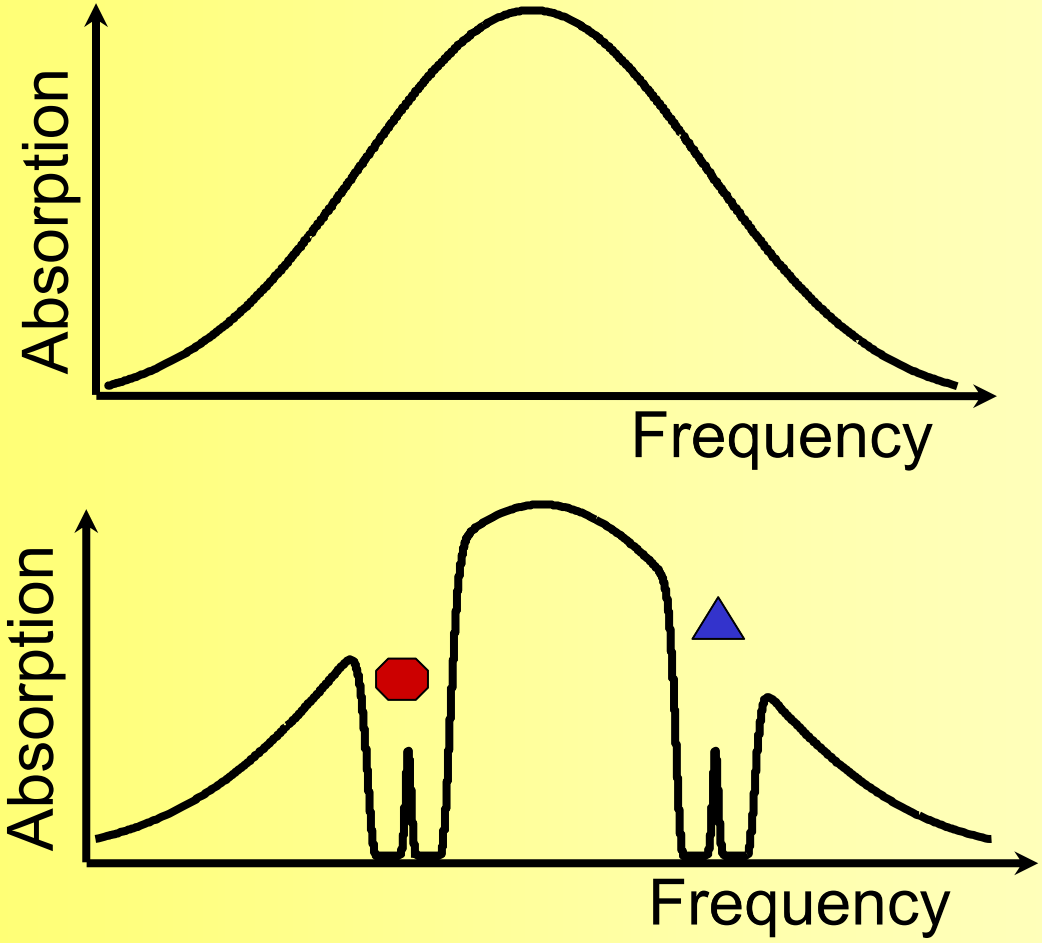




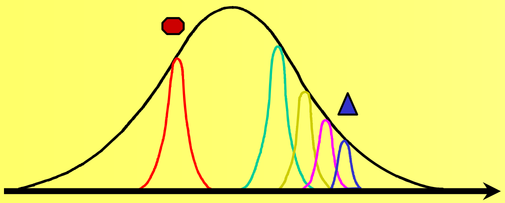
● ion absorbing at frequency ν_1

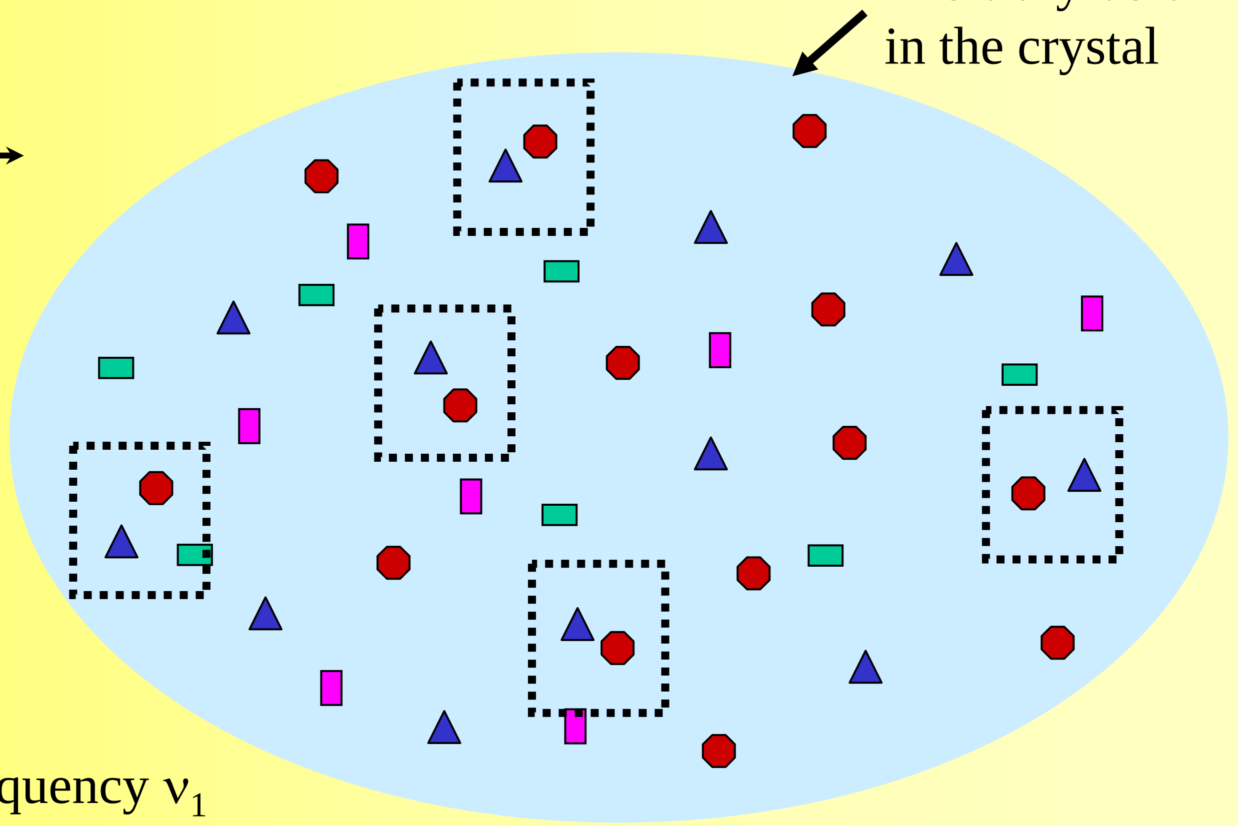
▲ ion absorbing at frequency ν_2

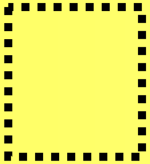
Inhomogeneous absorption profile is tailored to create qubit structures



Qubit distillation



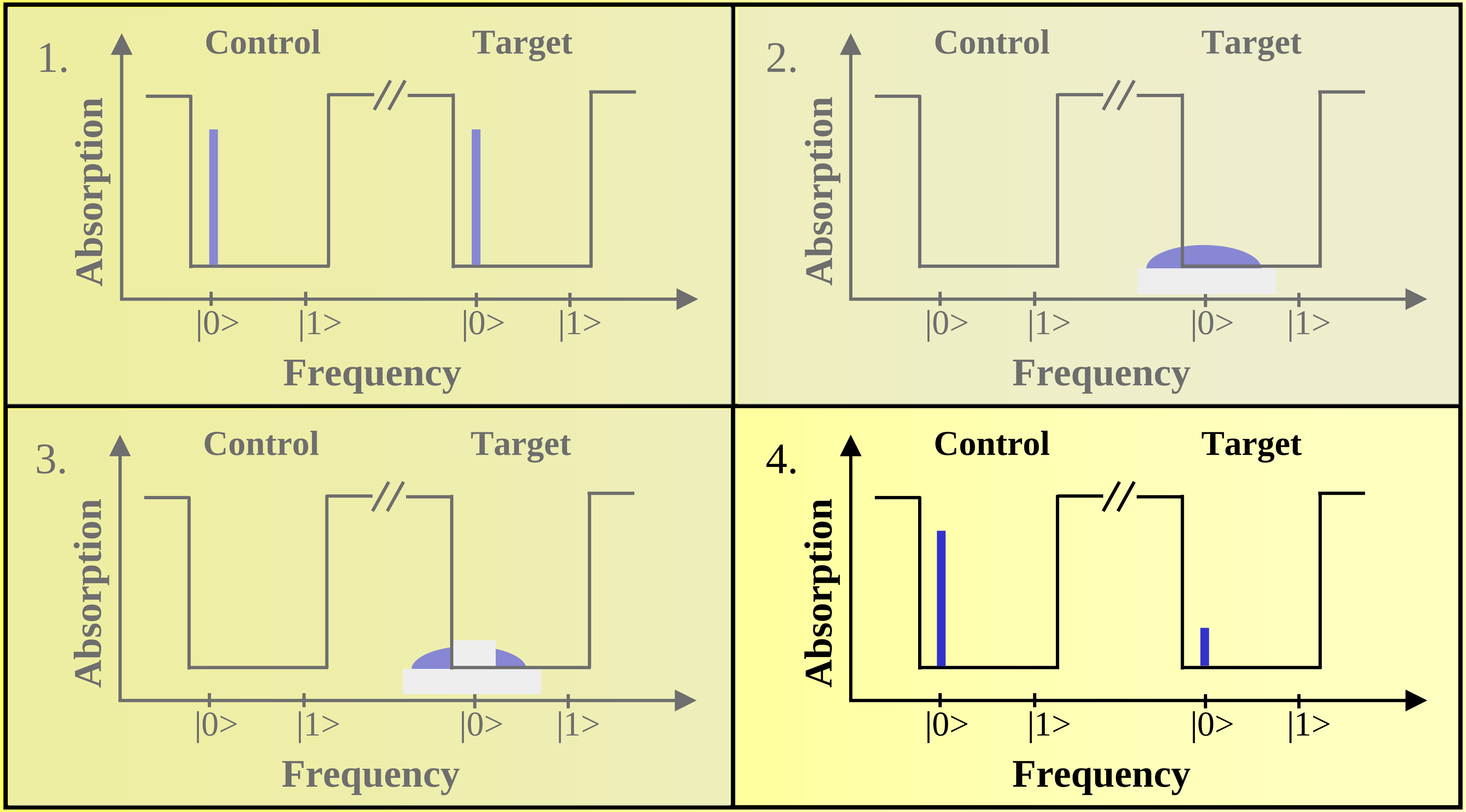




Ions interacting strongly enough for mutual control. Potential QC

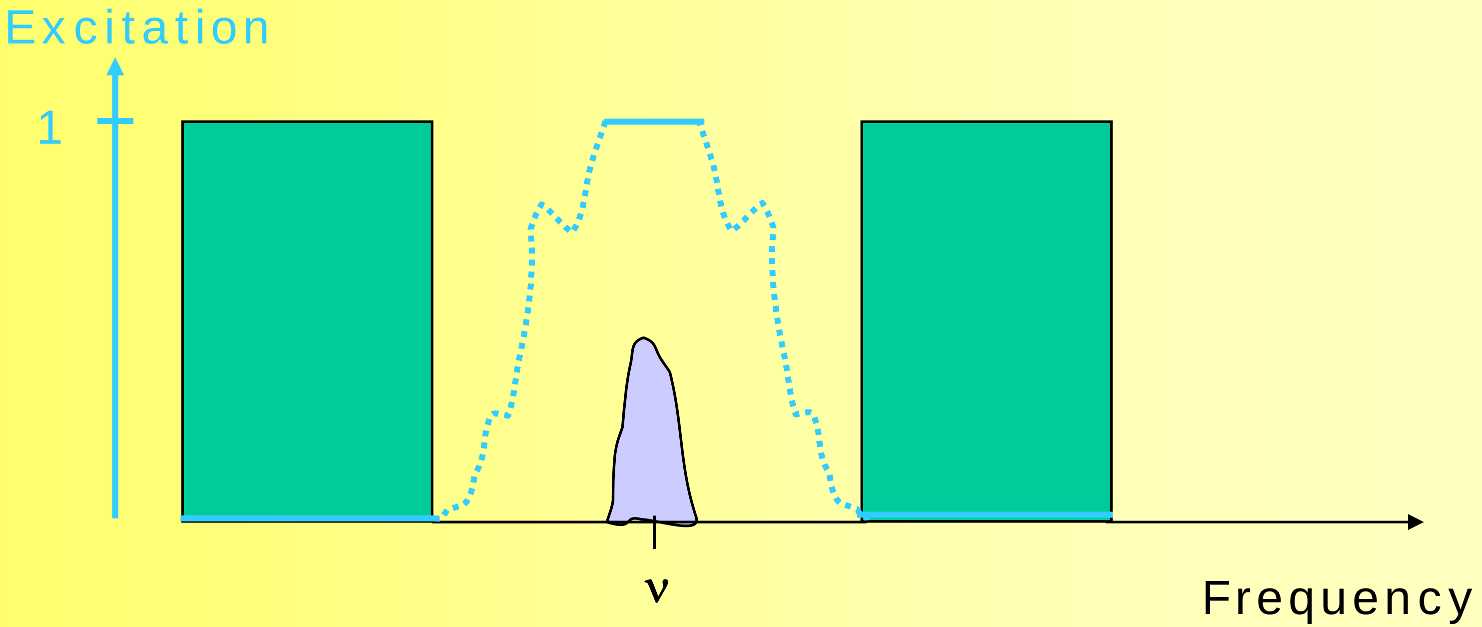
- ion absorbing at frequency ν_1
- ion absorbing at frequency ν_2

Selecting strongly interacting ions

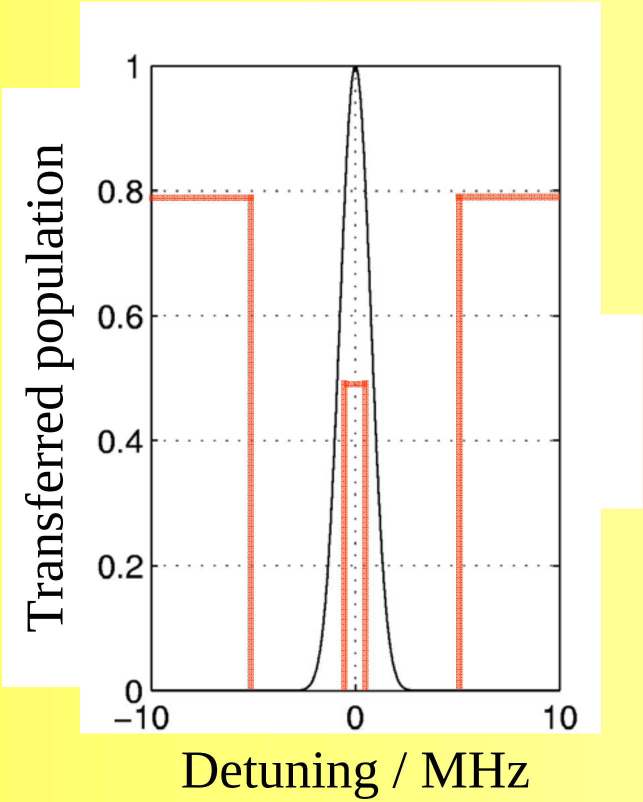


Requirements on excitation pulses

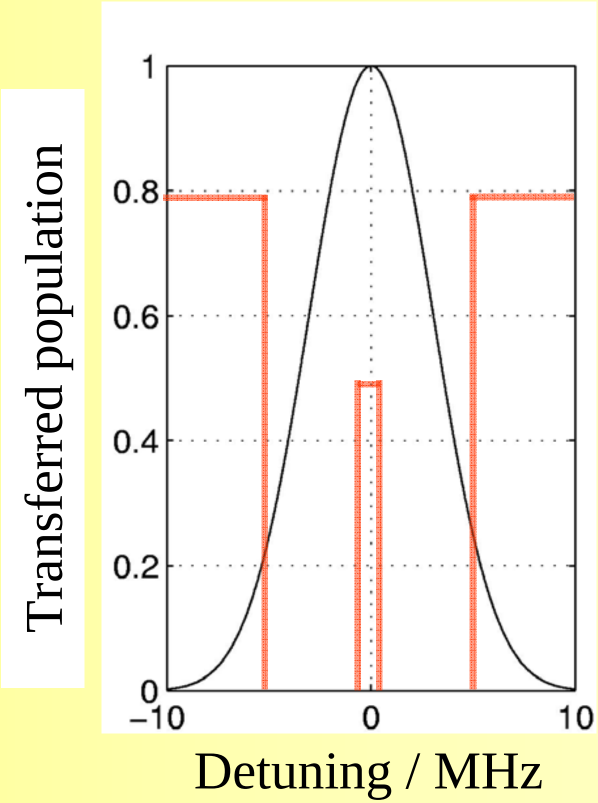
How to interact with the qubit ions without interacting with ions at nearby absorption frequencies



Excitation with Gaussian π-pulses



1 μs pulse



0.25 μs pulse

Pulse shapes for coherent transfer of population

This work was carried out by Ingela Roos together with Klaus Mølmer

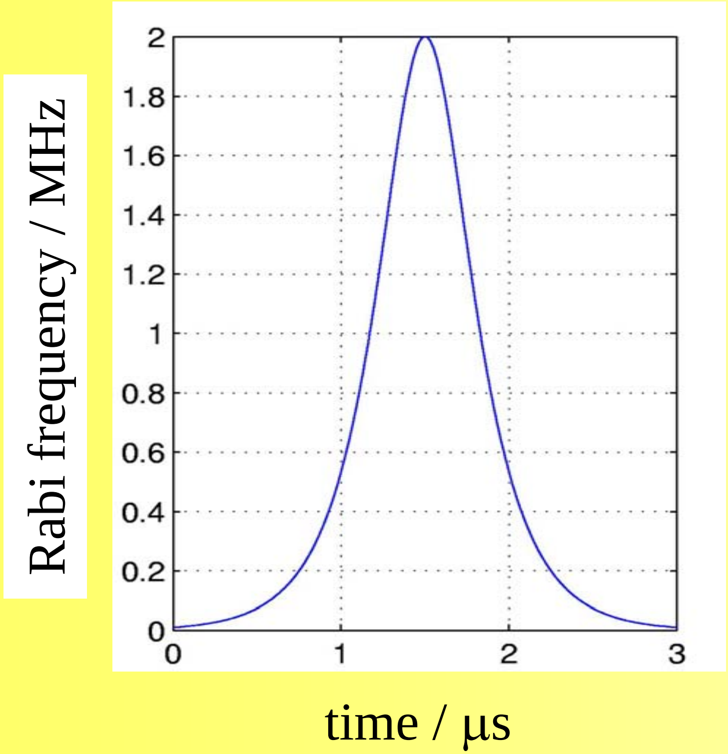
- "Robust quantum computing with composite pulse and coherent population trapping", Phys Rev A**69**, 22321 (2004)

Requirements

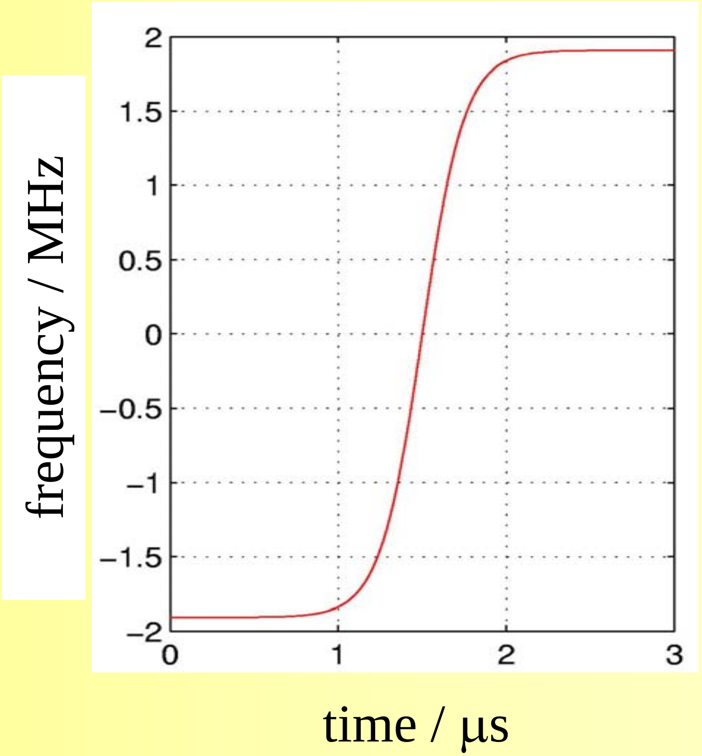
- Complete transfer of the peak of ions
- No excitation of surrounding ions

Complex hyperbolic secant pulse

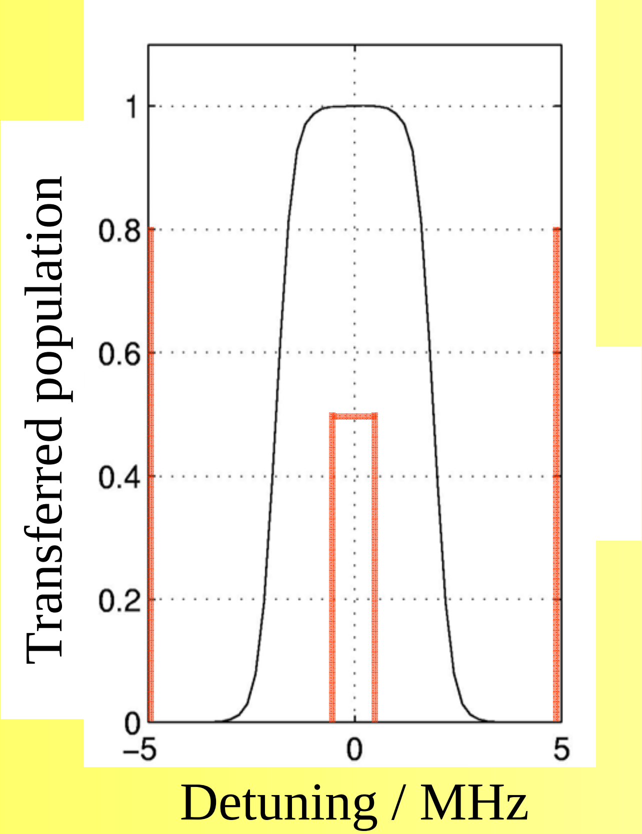
real sechyp amplitude envelope



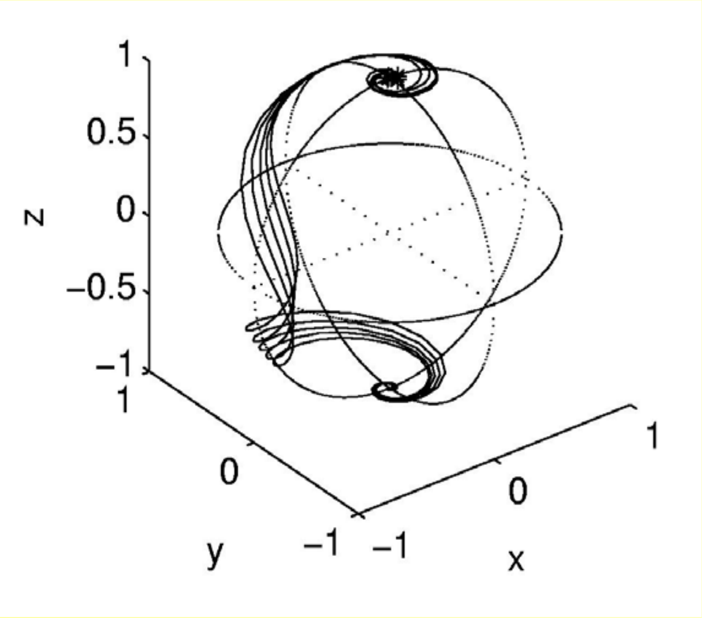
tanhyp frequency chirp



Excitation with complex hyperbolic secant pulse



Evolution on the Bloch sphere

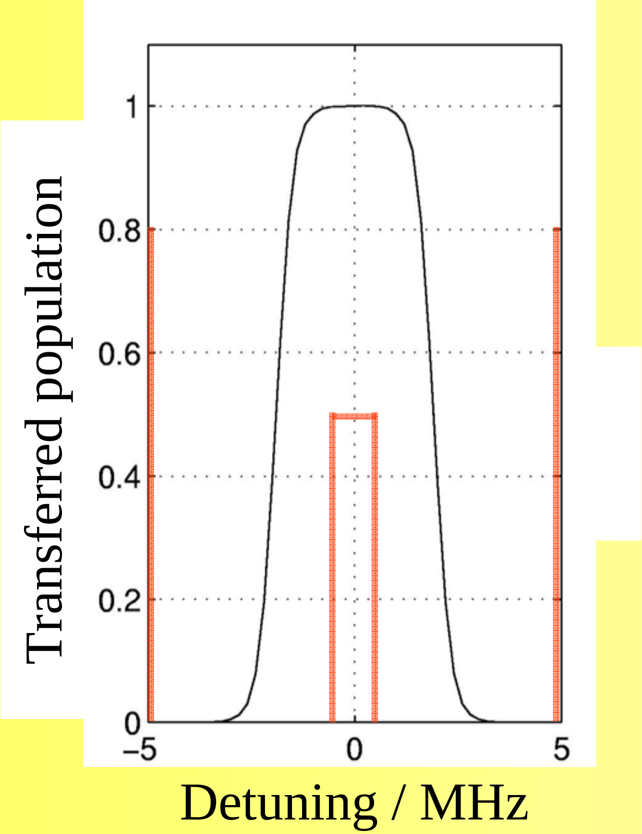


Further more, above a certain threshold intensity the operation is insensitive to different ions having different Rabi frequencies

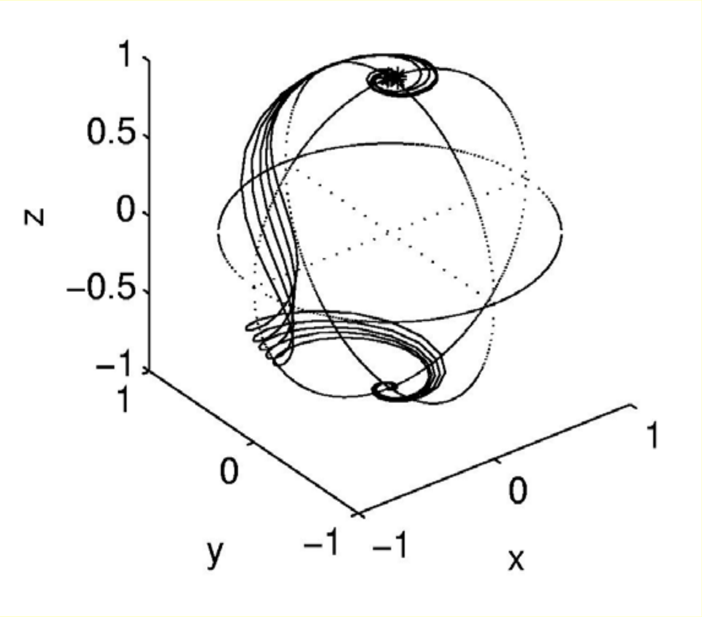
We must have same wave function for all QC instances

- Hyperbolic secant pulses can compensate for
  - Finite channel widths
    - Variation in detunings
  - Field inhomogeneities & ion orientation variations
    - Variations in laser coupling

Excitation with complex hyperbolic secant pulse



Evolution on the Bloch sphere



The right hand figure illustrates that the ions are driven coherently on the Bloch sphere

High fidelity qubit operations require coherent laser systems

- Rare earth ion coherence times
  - Pr:YSO, optical ~100 μs, qubit ~100 ms
  - Eu:YSO, optical ~1 ms, qubit ?
  - Er:YSO, optical (1.5 μm) several ms
- Dye lasers
  - Commercial, coherence time <1 μs
  - Dortmund system coherence time ~10 μs
  - Lund system coherence time ~100 μs
    - Drift <1 kHz/s

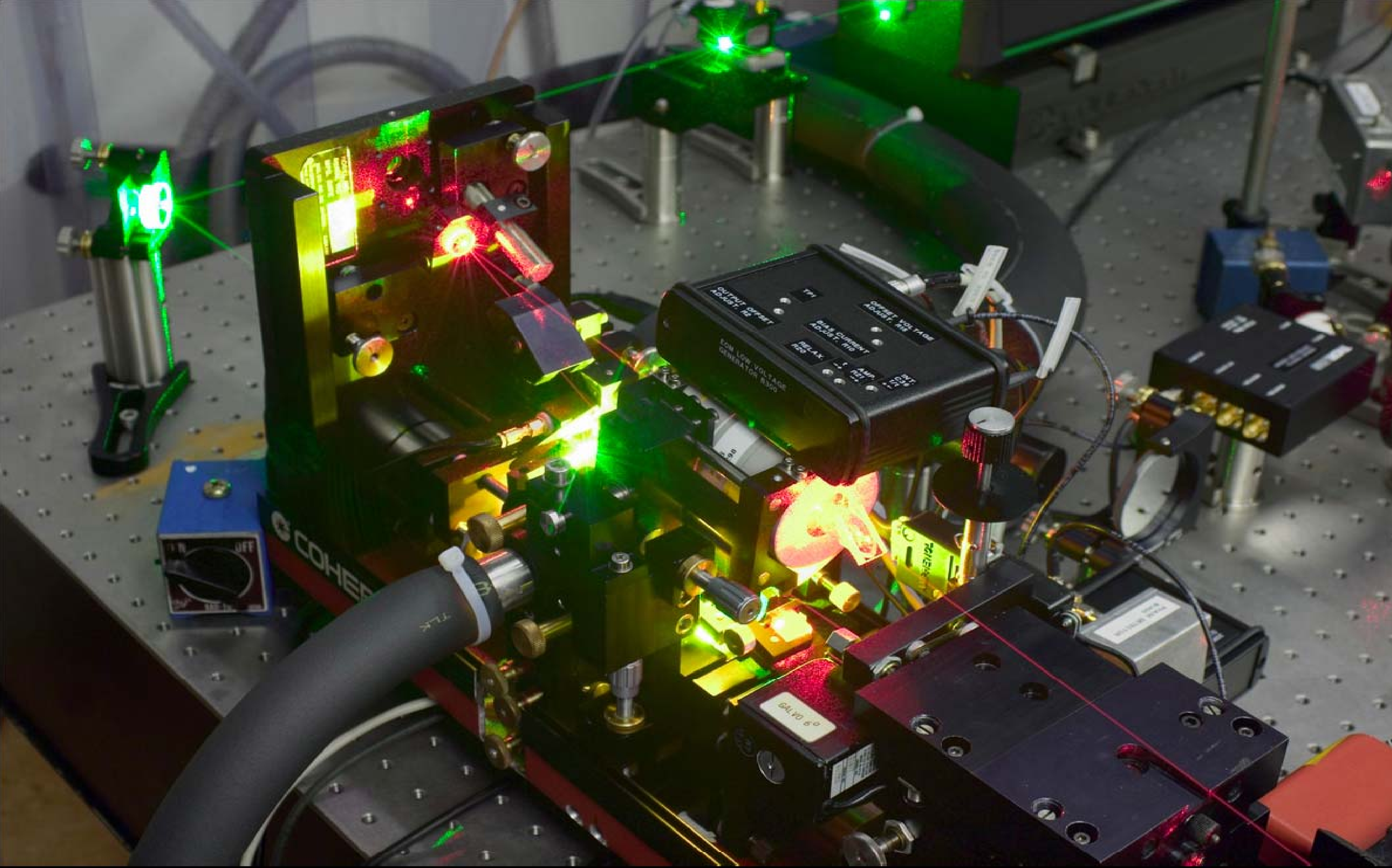
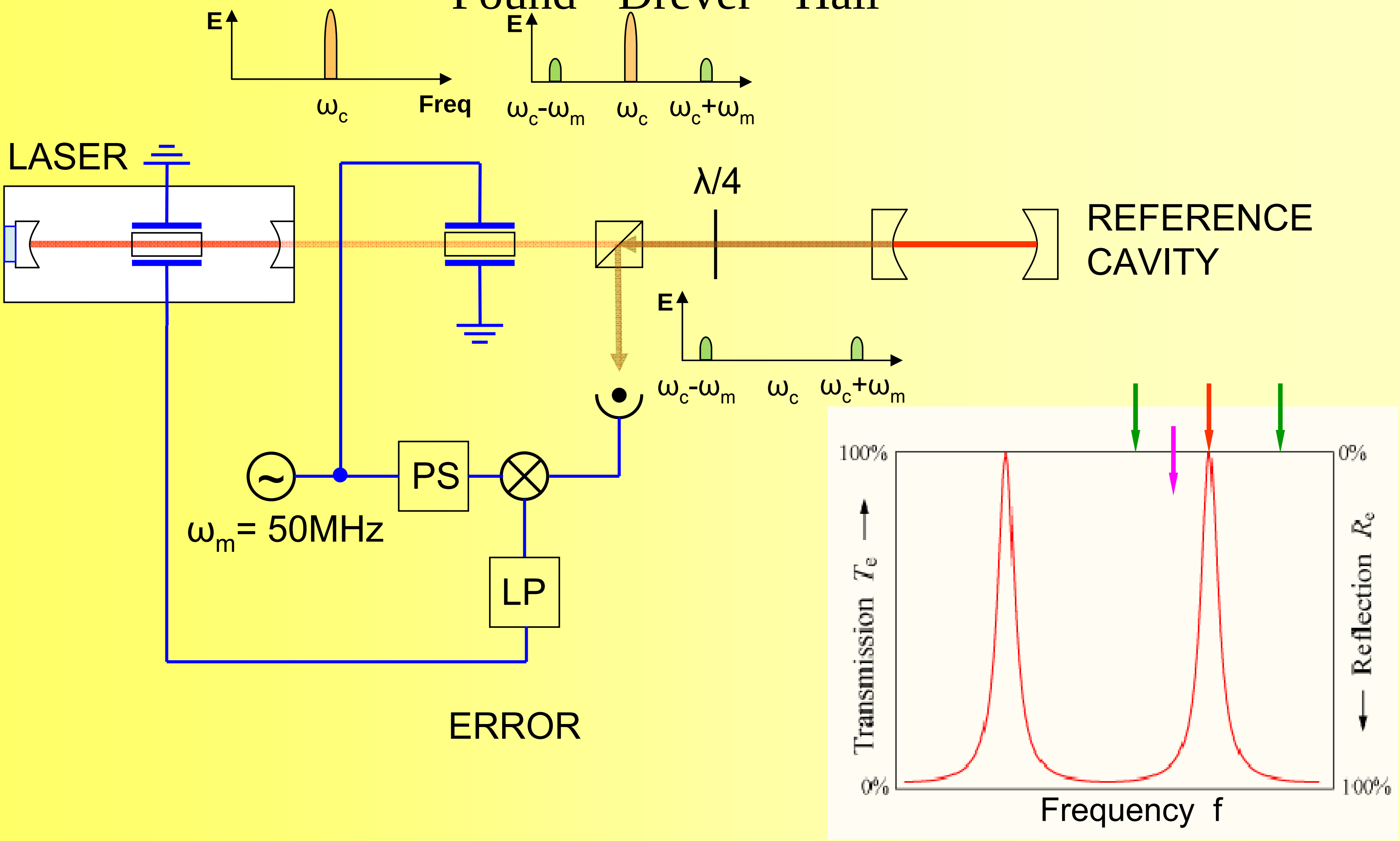


Photo: Tomas Svensson

Photo:
Tomas Svensson

Phase stabilization against a cavity

Pound - Drever - Hall

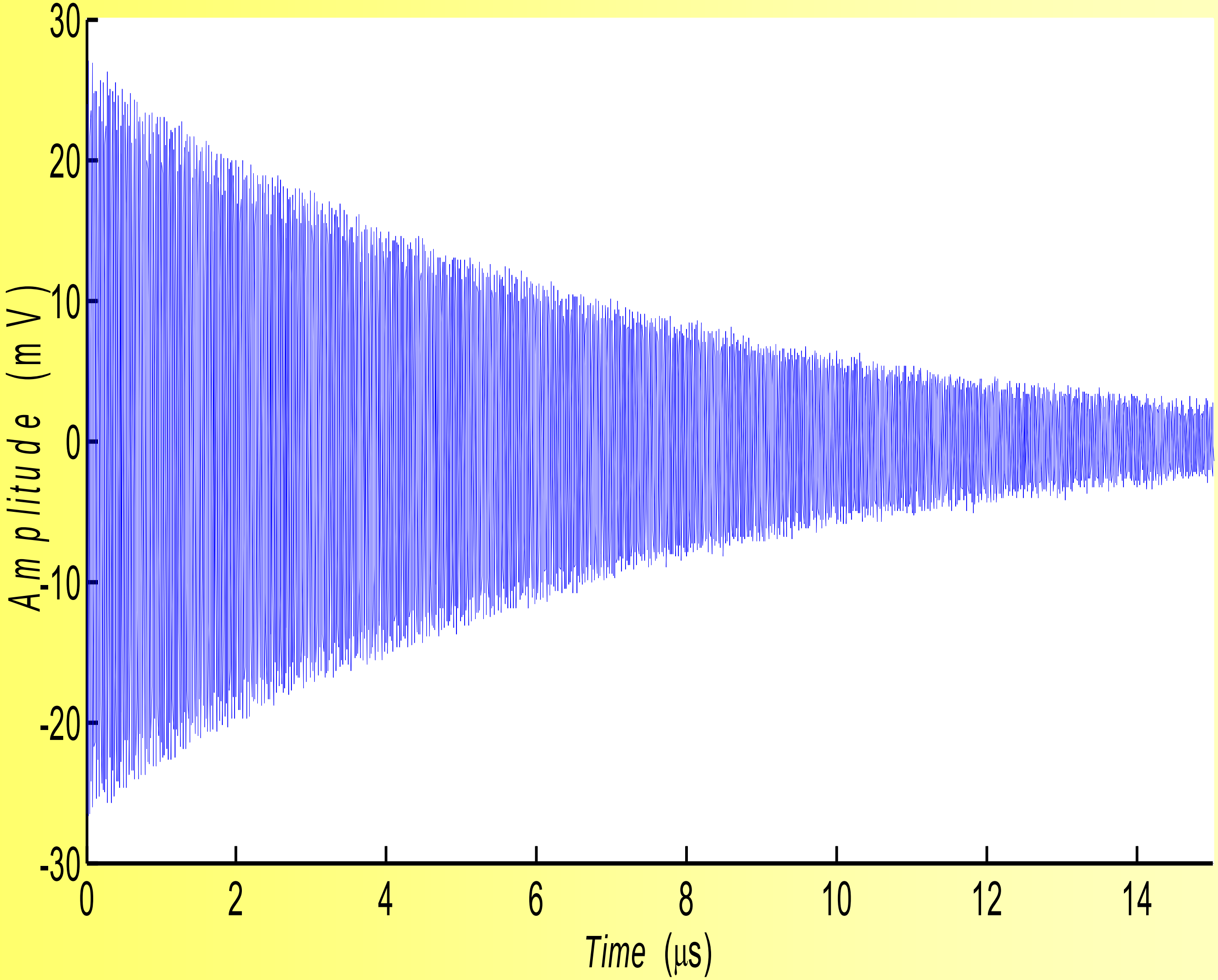


Locking to spectral hole

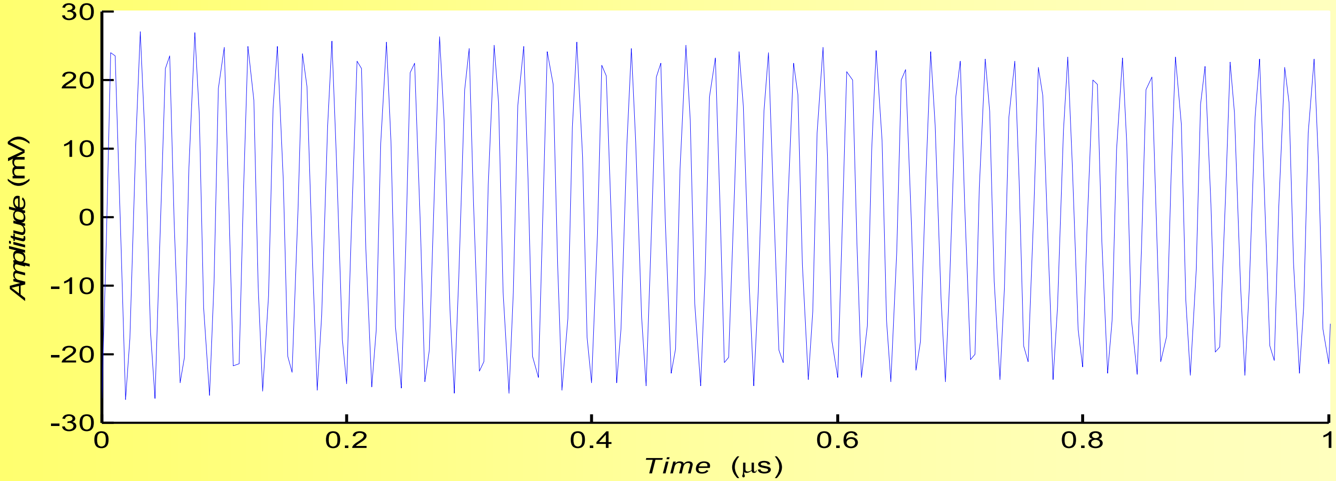
- Cavity with fixed transmission linewidth is replaced against a spectral hole with a linewidth that dynamically adapts to the laser linewidth
- Different optimum modulation index
- Optimum absorption
- New locking regimes
  - system locks leading to constant drift

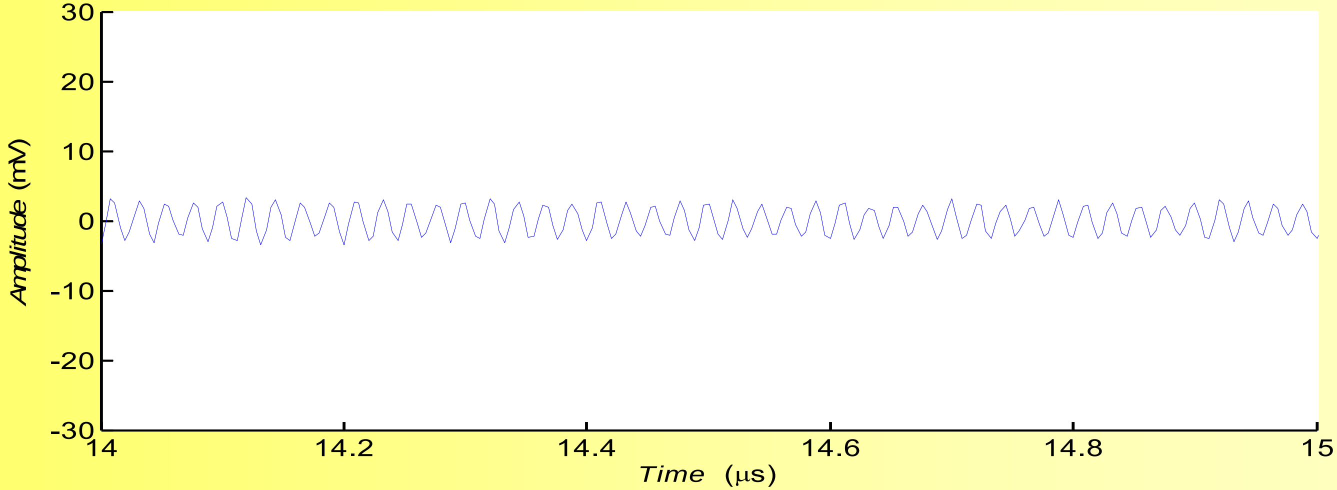
Free induction decay Pr:YSO

Beat signal between laser and Pr ions

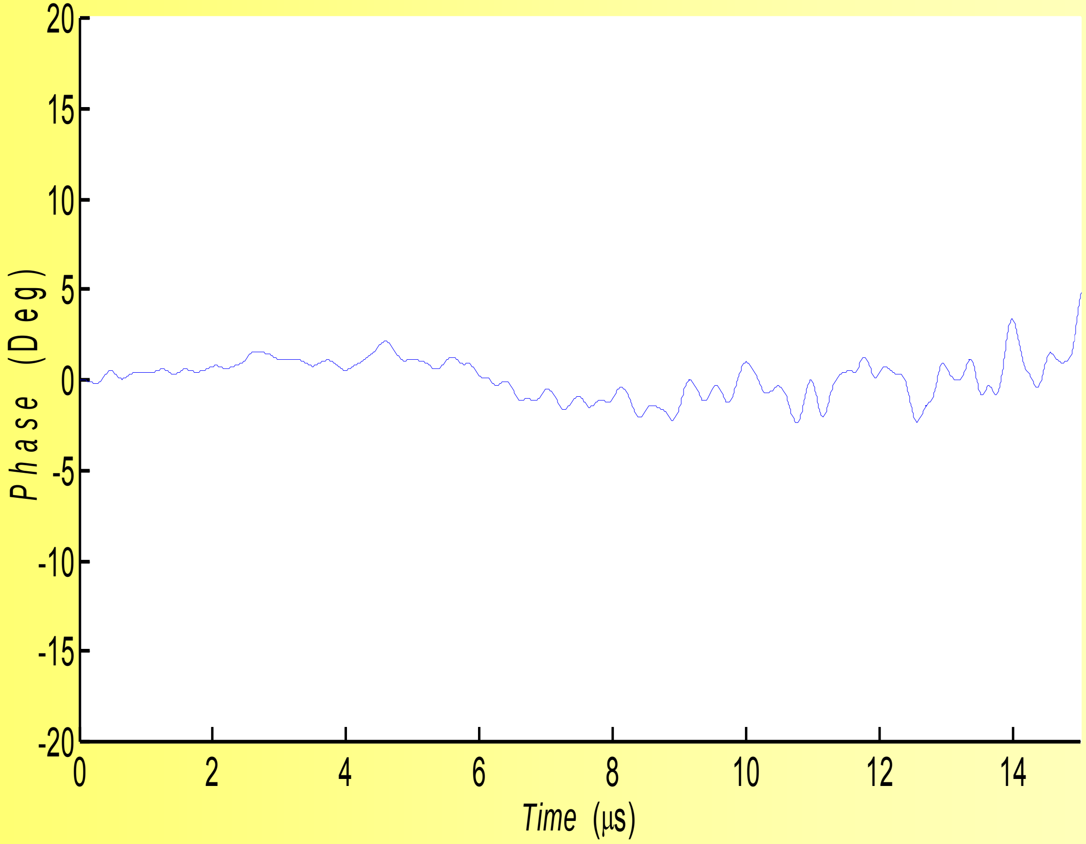


Free induction decay



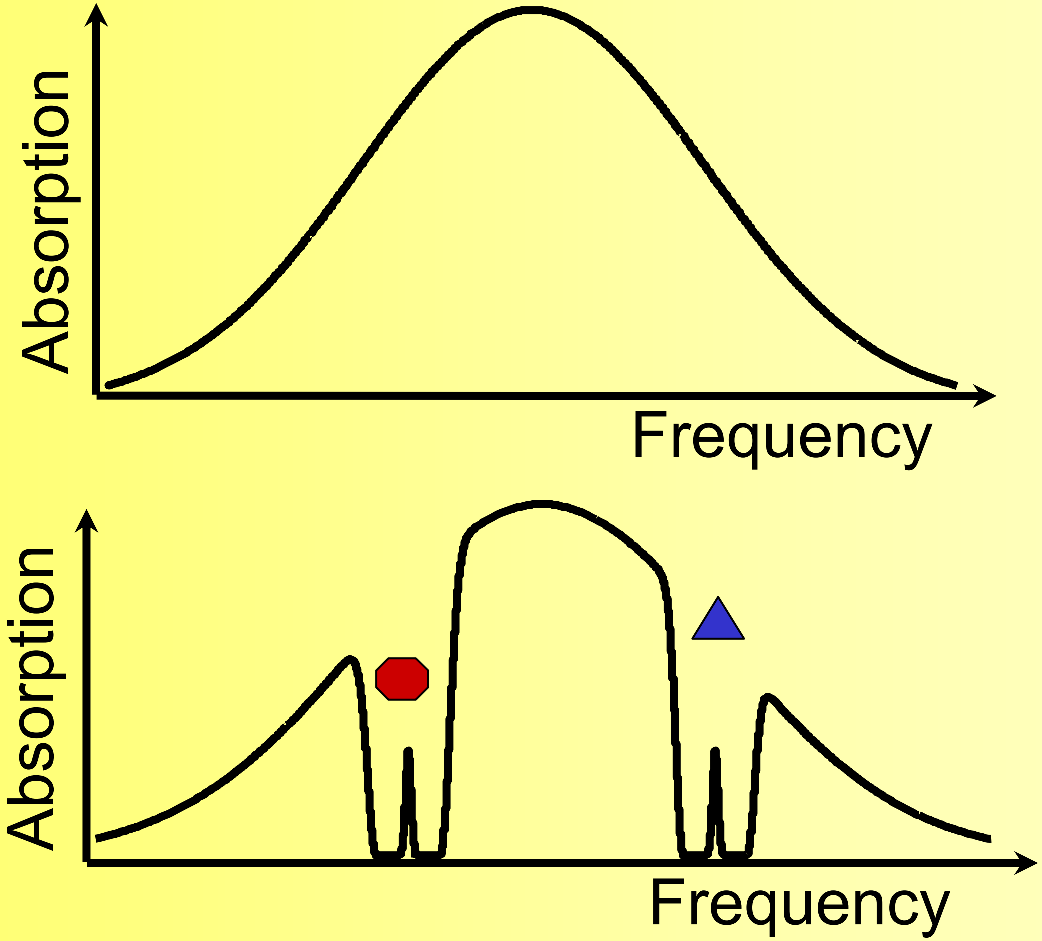


Laser phase drift <5° over 10 μs
Coherence time > 100 μs

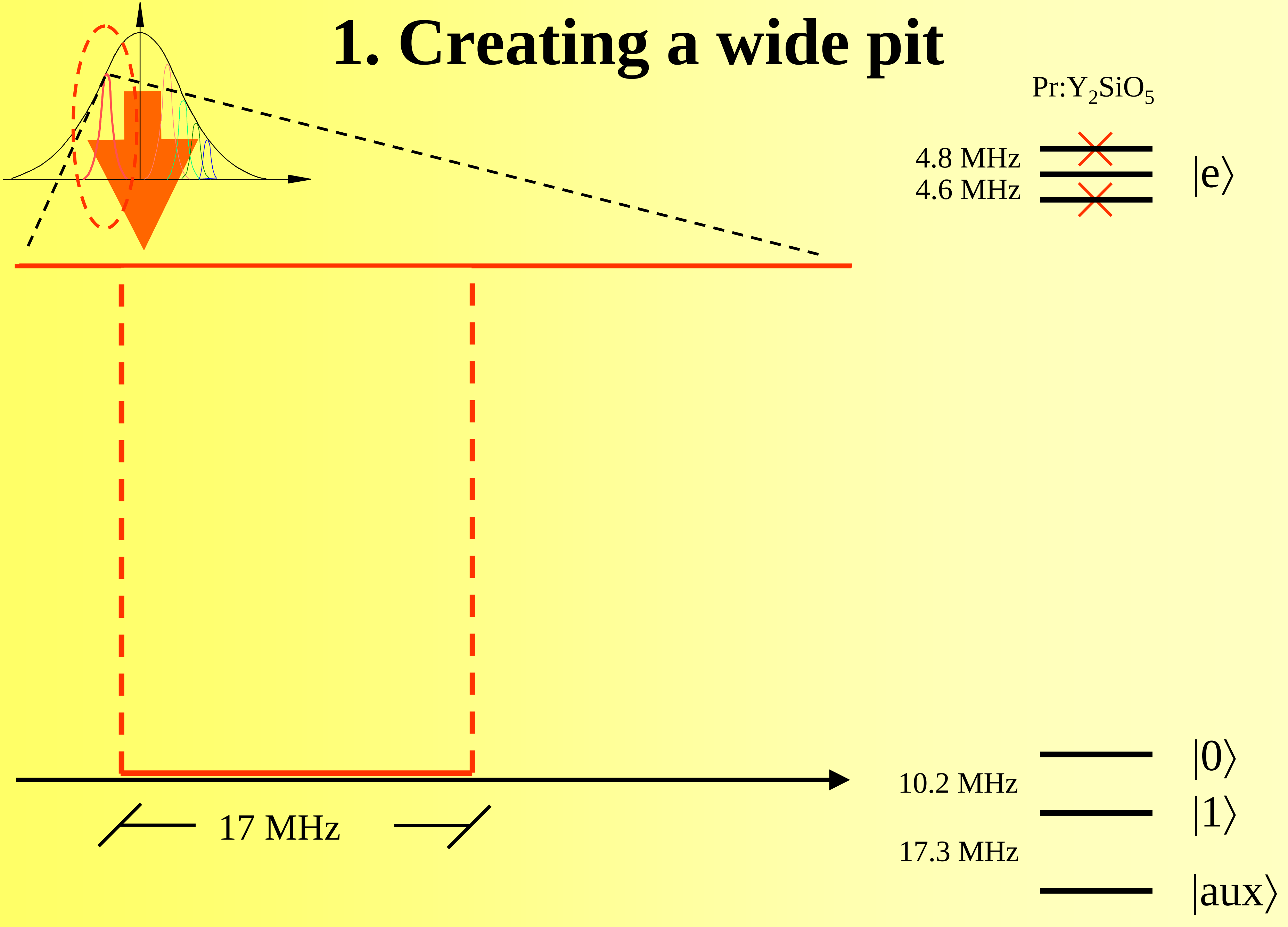


Qubit creation

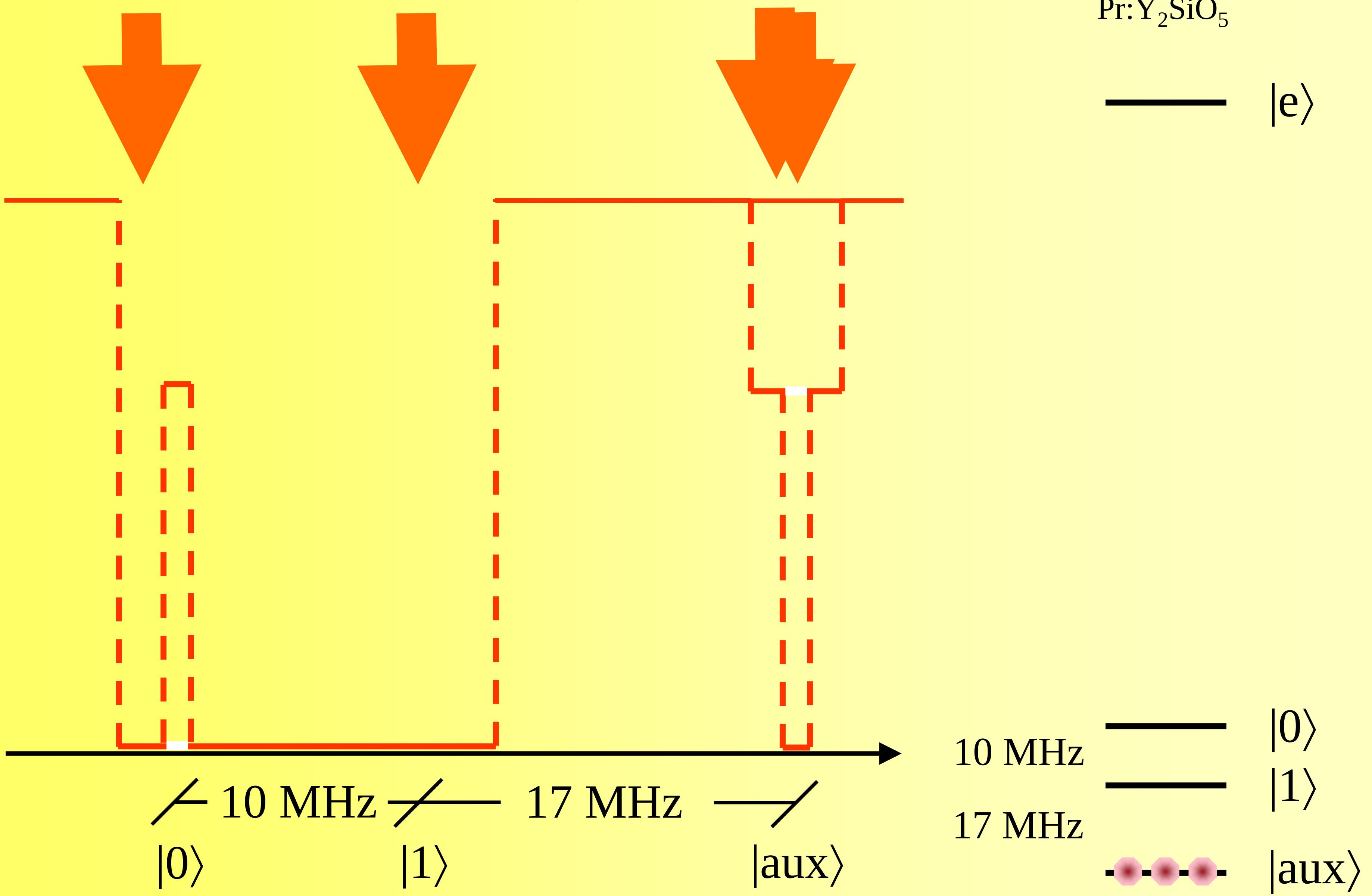
Inhomogeneous absorption profile is tailored to create qubit structures



1. Creating a wide pit

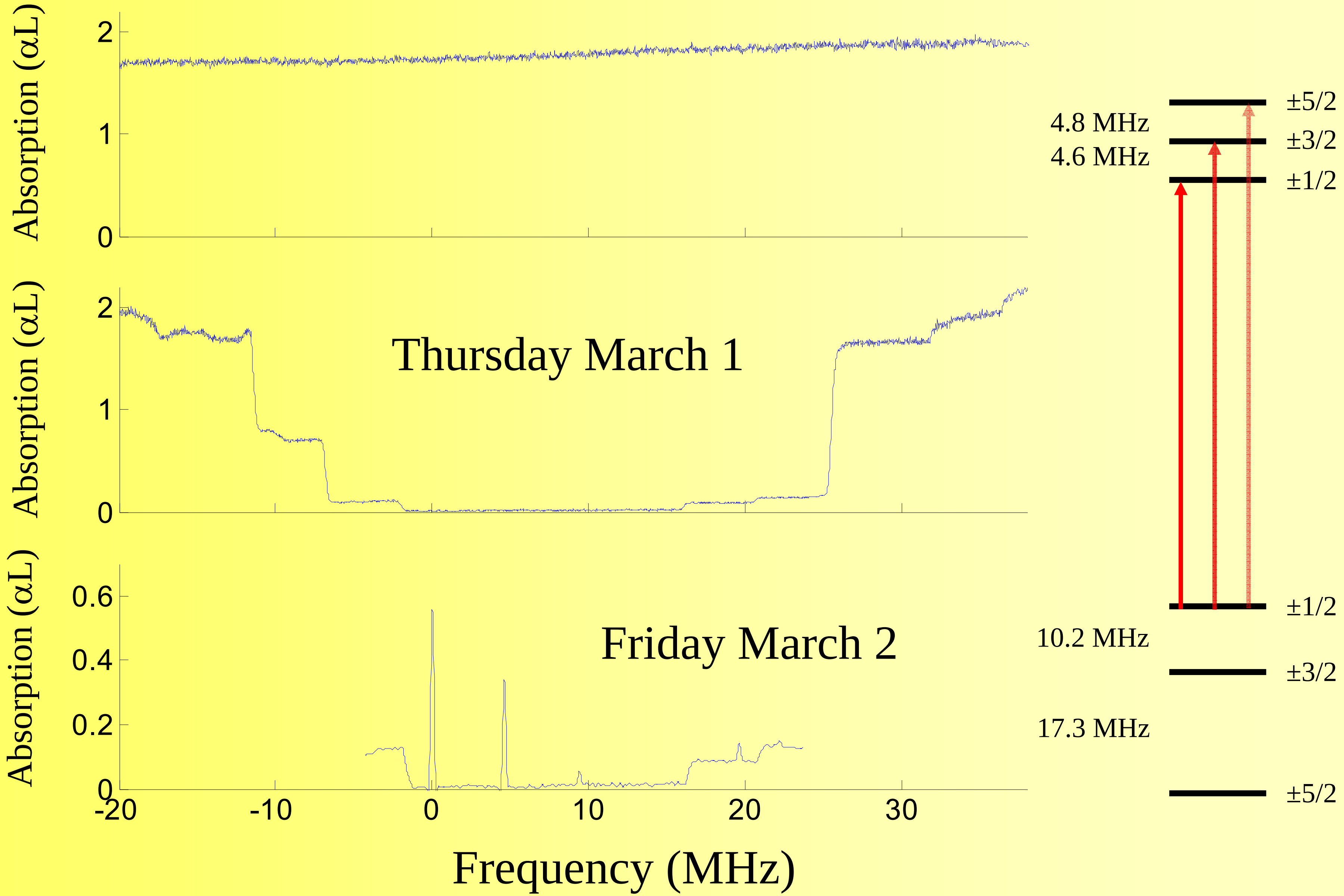


2. Isolating one type of ions

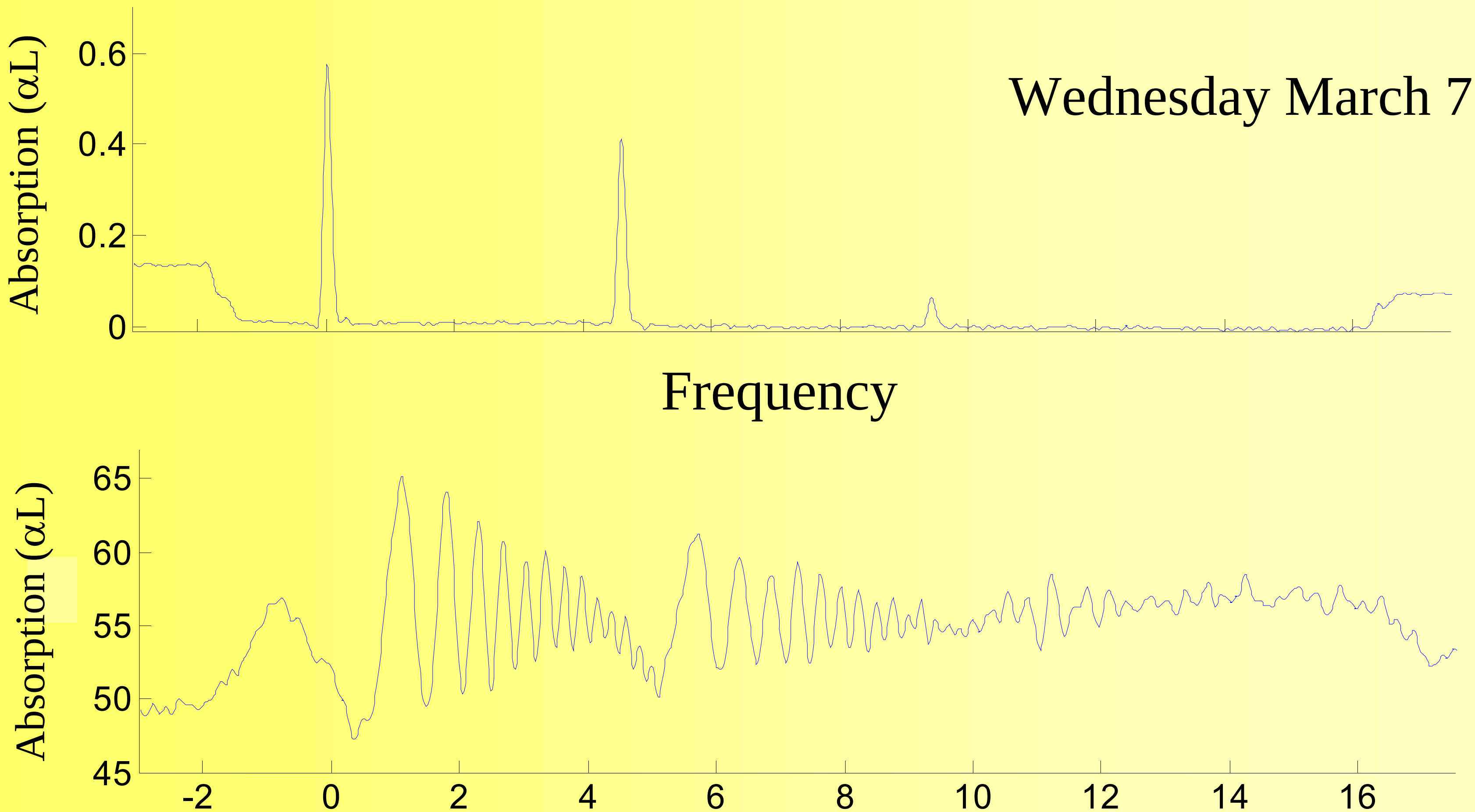


Experimental data

Pit & peak creation



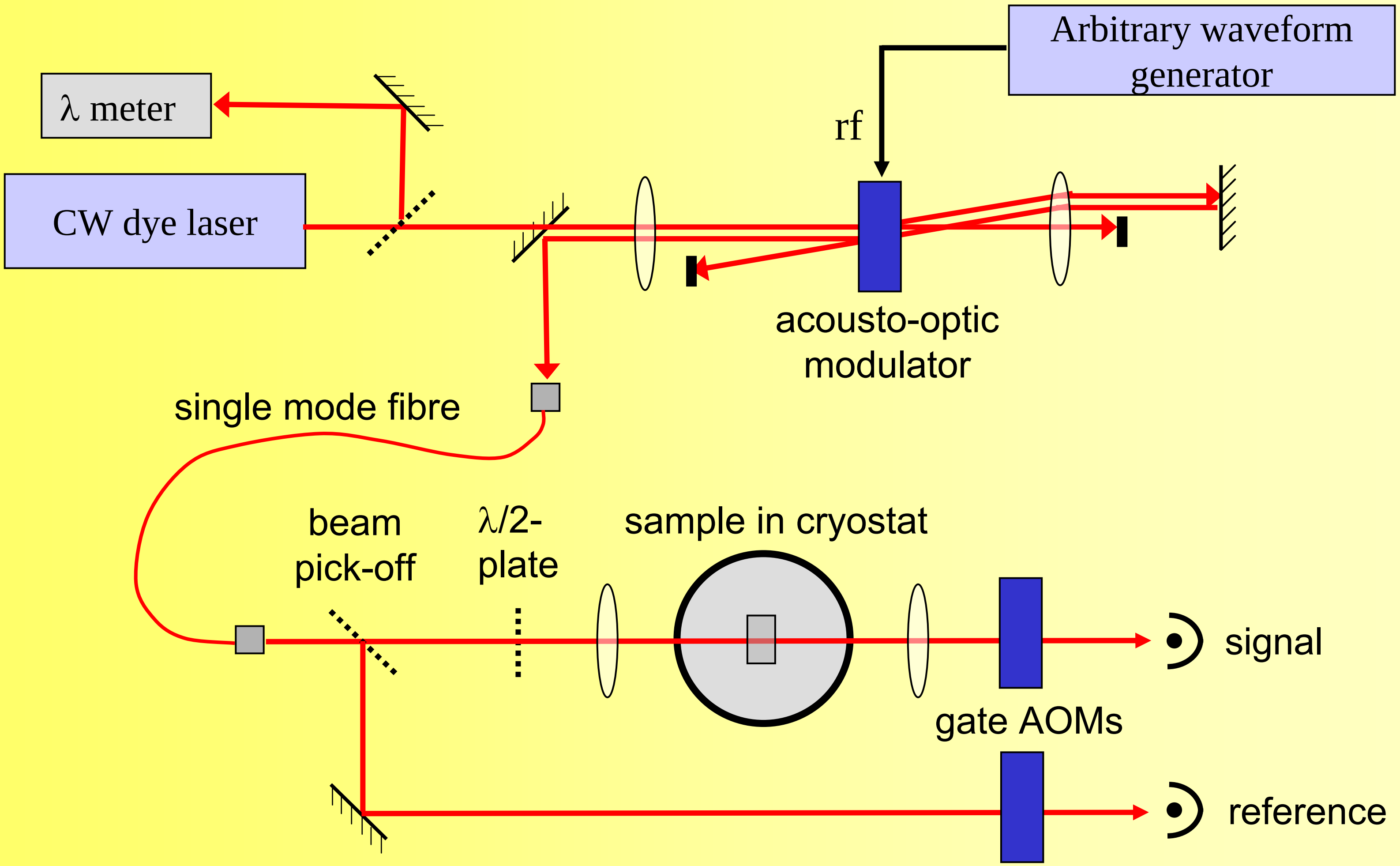
Readout procedure



Frequency chirp rate 1MHz/μs

Time (μs)
Frequency (MHz)

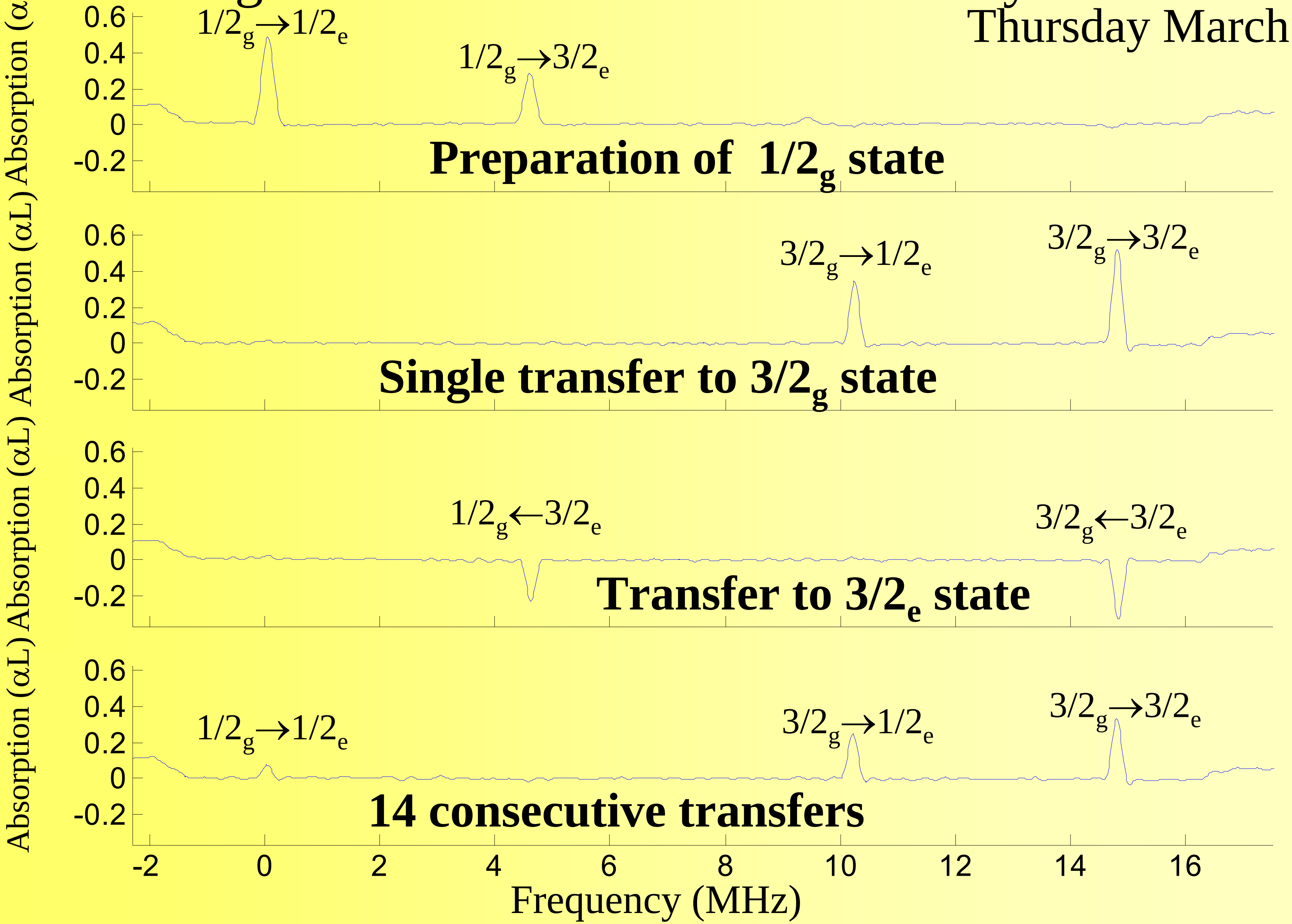
Experimental set-up



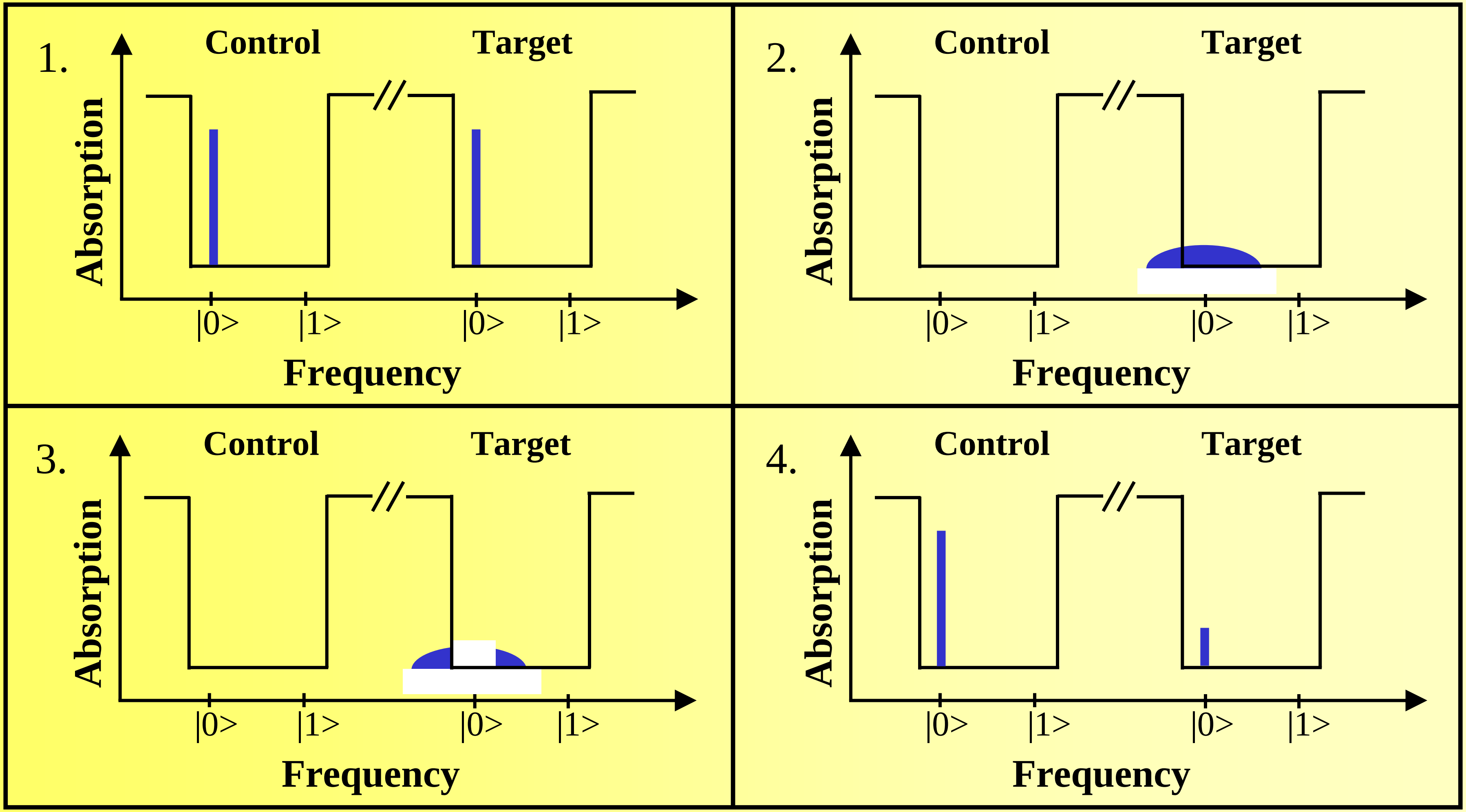
State to state transfer

Single state-to state transfer efficiency 97.5%

Thursday March 8

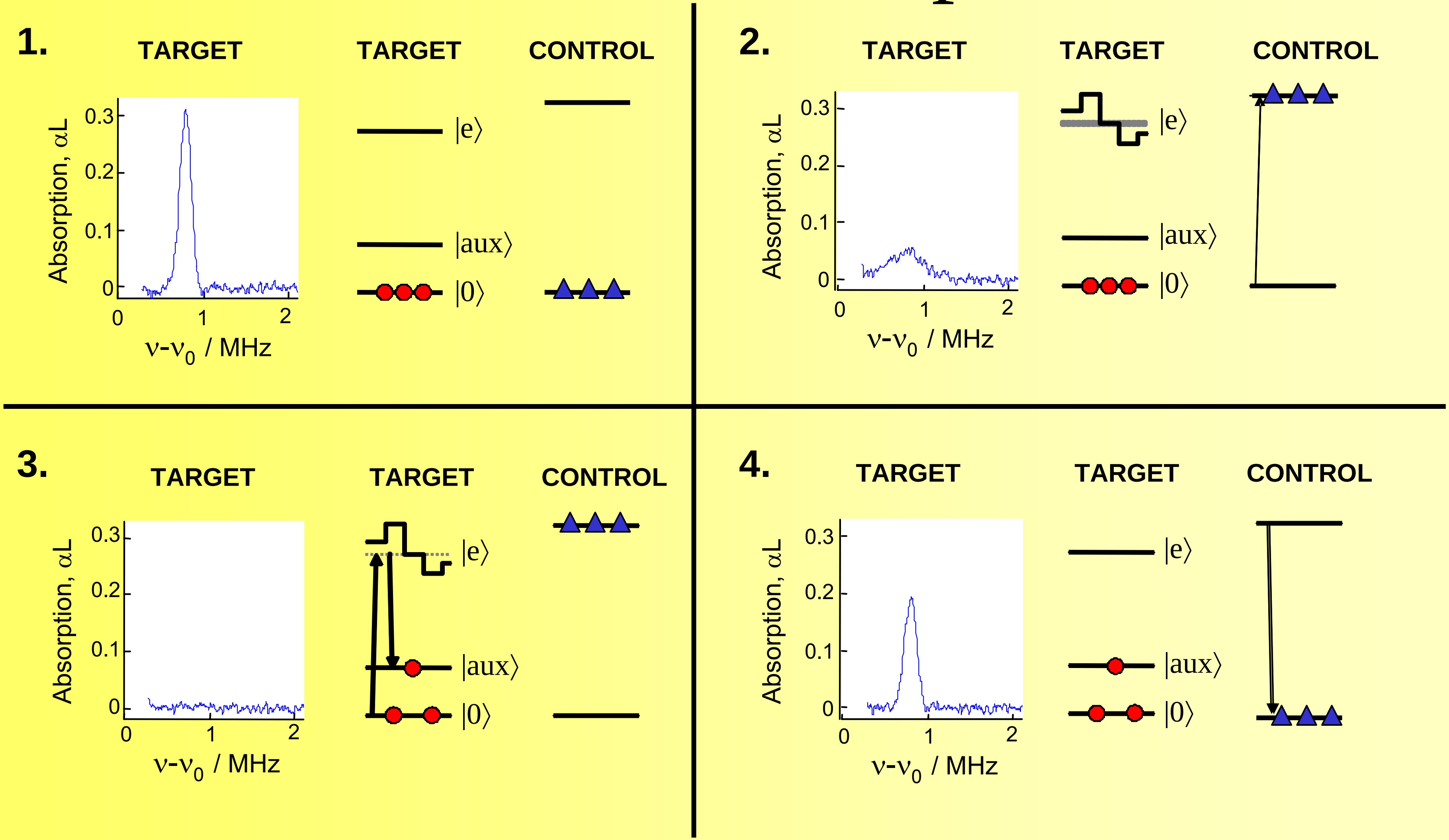


Selecting strongly interacting ions



With Dieter Suter and Robert Klieber in Dortmund

Qubit distillation experiment



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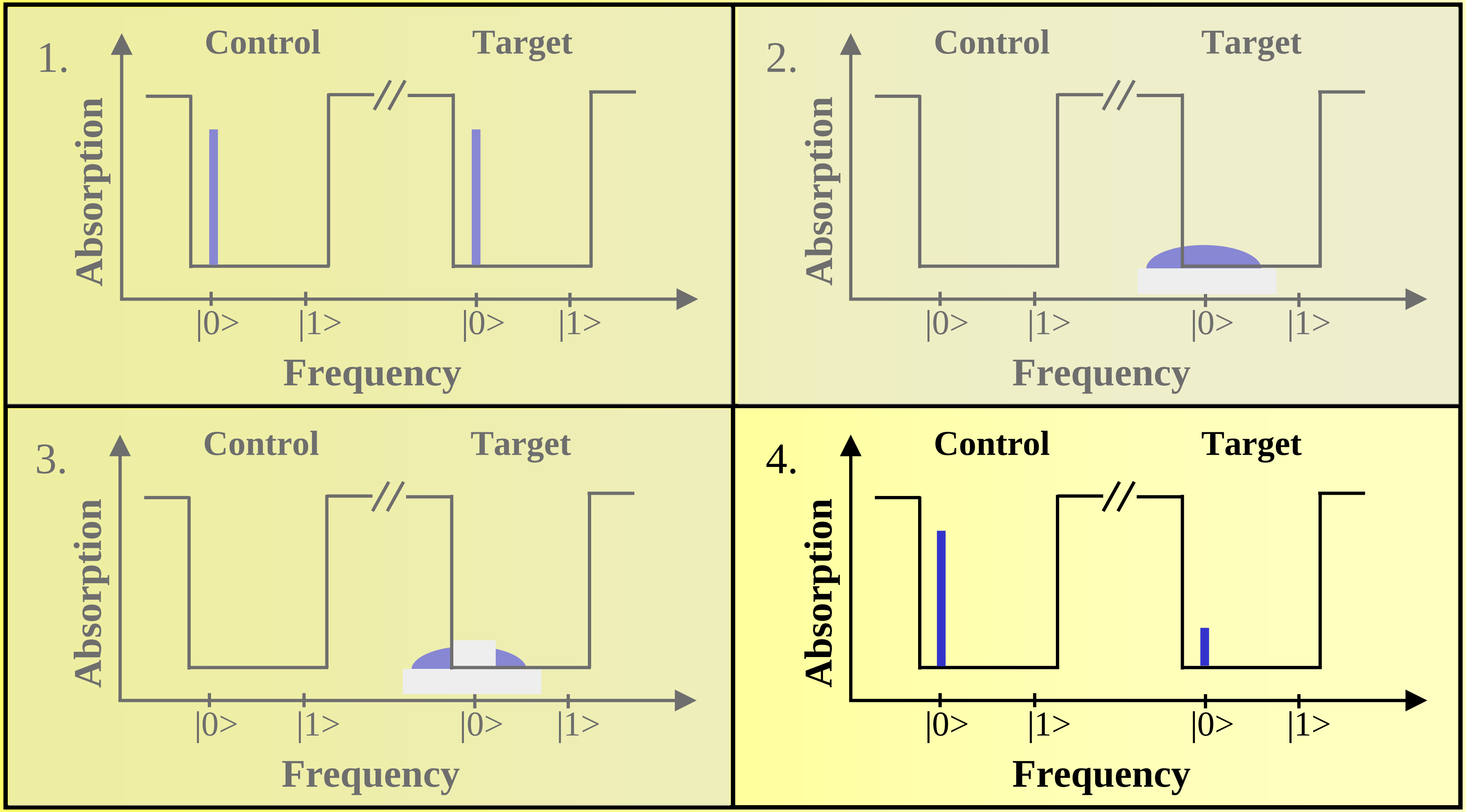
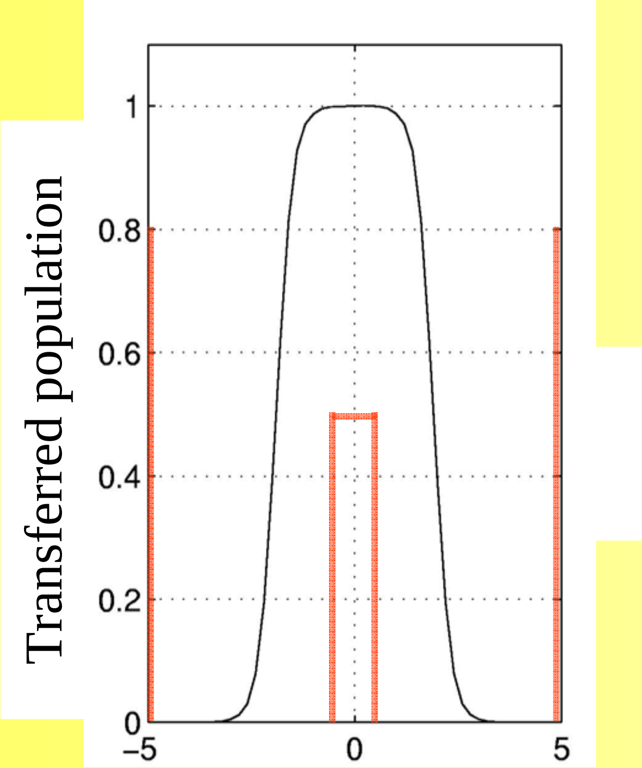


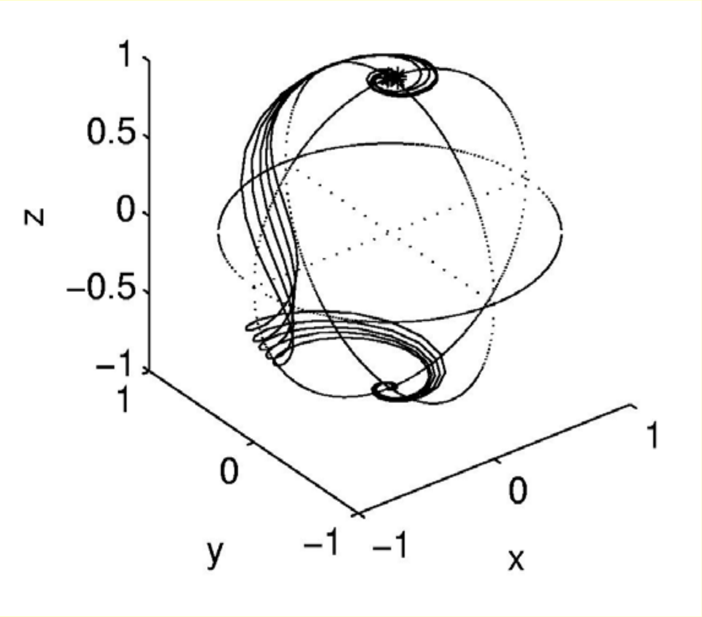
Arbitrary single qubit operations

- Secant hyperbolic pulses compensate for the inhomogeneous broadening in the qubit for transitions going from one pole to the other pole on the Bloch sphere
- Thus they are not readily applicable to arbitrary single qubit operations

Excitation with complex hyperbolic secant pulse



Evolution on the Bloch sphere



Further more, above a certain threshold intensity the operation is insensitive to different ions having different Rabi frequencies

Pulse sequences for arbitrary single qubit operations

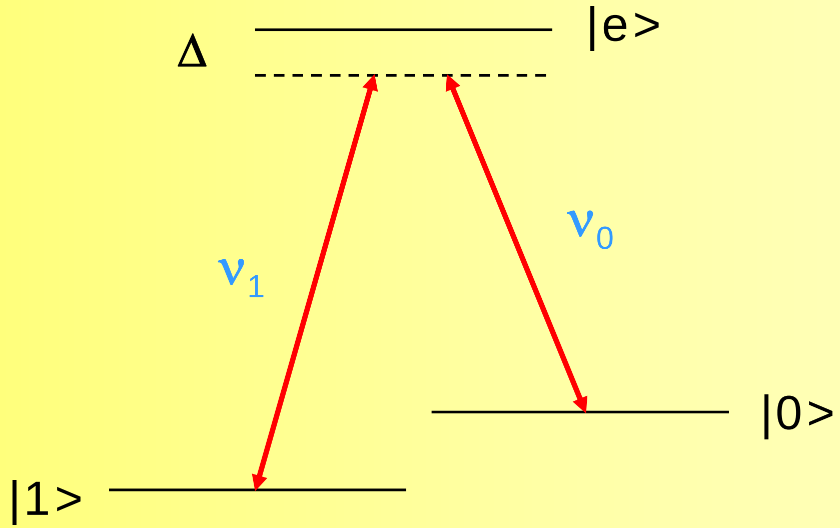
This work was carried out by Ingela Roos together with Klaus Mølmer

- "Robust quantum computing with composite pulse and coherent population trapping", Phys Rev A**69**, 22321 (2004)

Requirements

- Find pulses compensating for instances having different transition frequencies and different ion-field coupling strength

Three-level system



$$\dot{c}_e = -i\Delta c_e + i\frac{\Omega_{R0}(t)}{2}e^{-i\varphi_0}c_0 + i\frac{\Omega_{R1}(t)}{2}e^{-i\varphi_1}c_1$$

Three-level system

$$\Omega_{R0} = \Omega_{R1} \equiv \Omega_R$$

$$\Rightarrow \dot{c}_e = -i\Delta c_e + i\frac{\Omega_R(t)}{2}\underbrace{\left(e^{-i\varphi_0}c_0 + e^{-i\varphi_1}c_1\right)}_{= 0}$$

$$\varphi_1 - \varphi_0 = \phi$$

Coherent trapping

$$|d\rangle = \frac{1}{\sqrt{2}}(|0\rangle - \exp(-i\phi)|1\rangle)$$ ⇒ |d> is a dark state

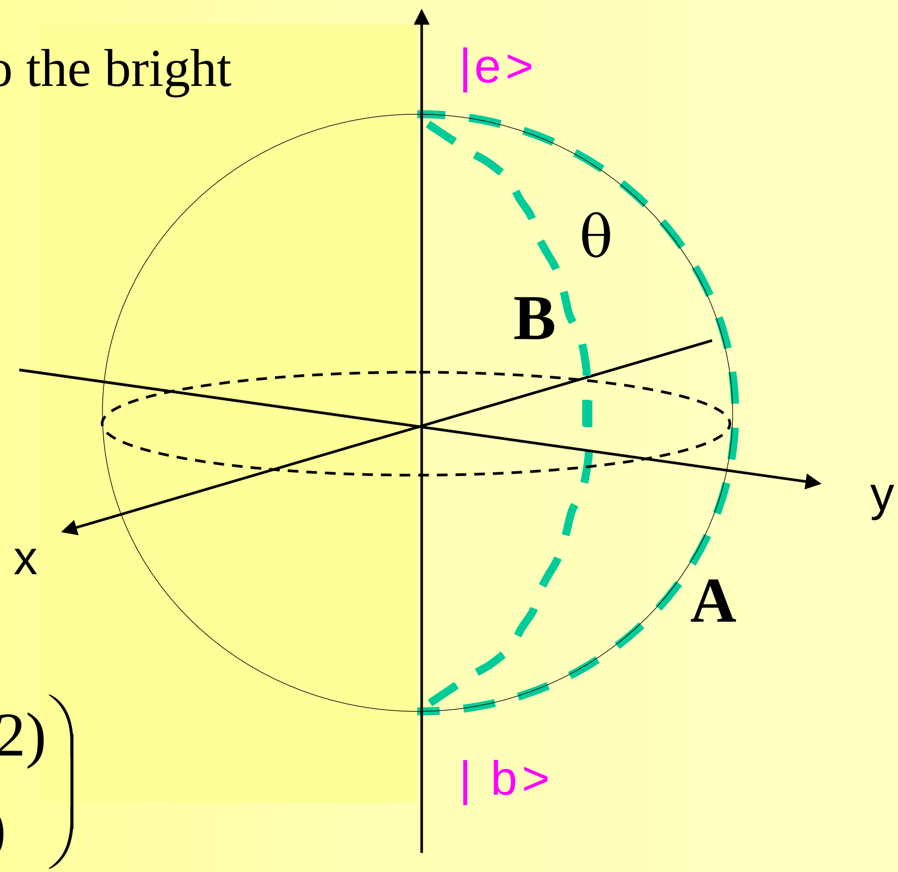
$$|b\rangle = \frac{1}{\sqrt{2}}(|0\rangle + \exp(-i\phi)|1\rangle)$$ ⇒ |b> is a bright state

Single qubit operations

unitary rotation about any axis on the equator of the Bloch sphere representing the qubit state basis

A two-colour sechyp pulse with phases φ_0 and φ_1 will drive the system into the excited state along route **A**.

A new two-colour sechyp pulse with phases $\varphi_0 + \pi + \theta$ and $\varphi_1 + \pi + \theta$ will drive the system to the bright state along route **B**.



This corresponds to the operation

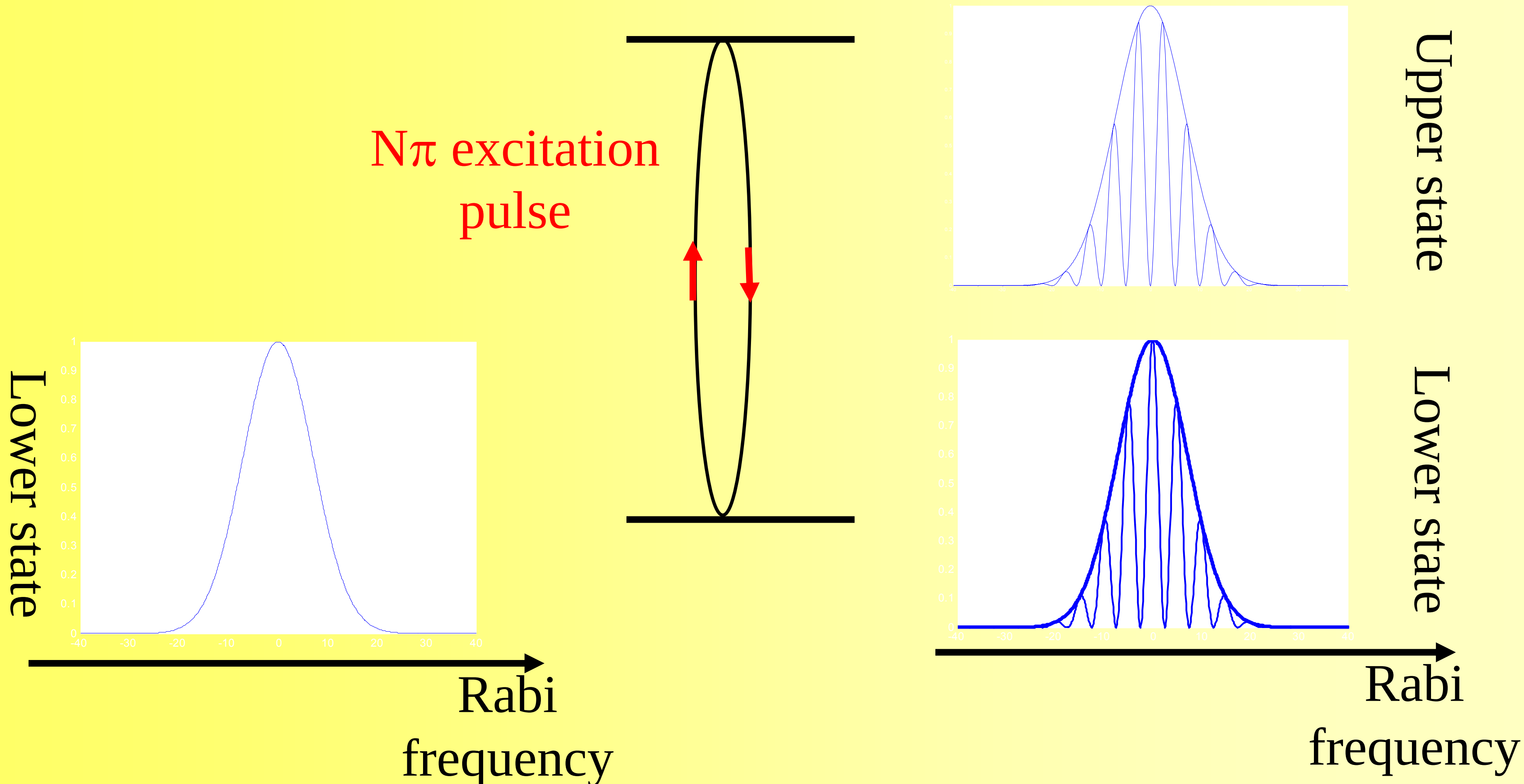
$$|b> \rightarrow e^{i\theta}|b>$$

Which in the original |0>, |1> basis corresponds to the unitary operation

$$U = e^{i\theta/2}\begin{pmatrix} \cos(\theta/2) & ie^{i\phi}\sin(\theta/2) \\ ie^{-i\phi}\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

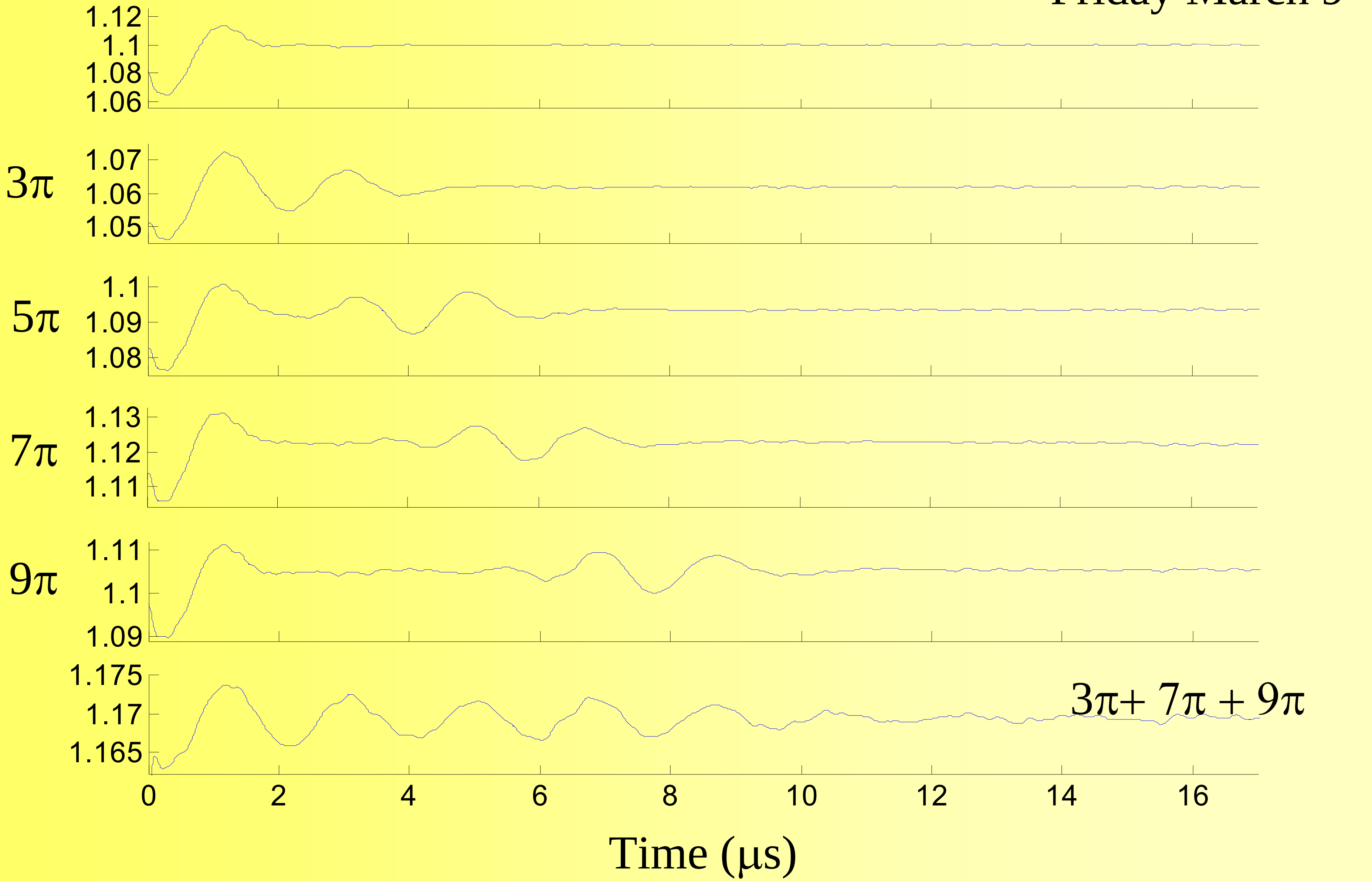
Rabi frequency selection

Ions in different parts of the beam experience different Rabi frequencies



Rabi frequency distillation

Friday March 9



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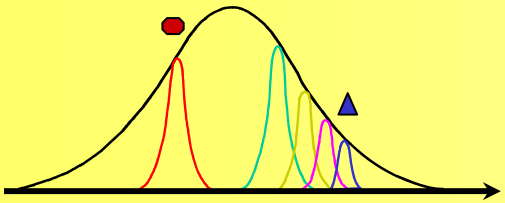
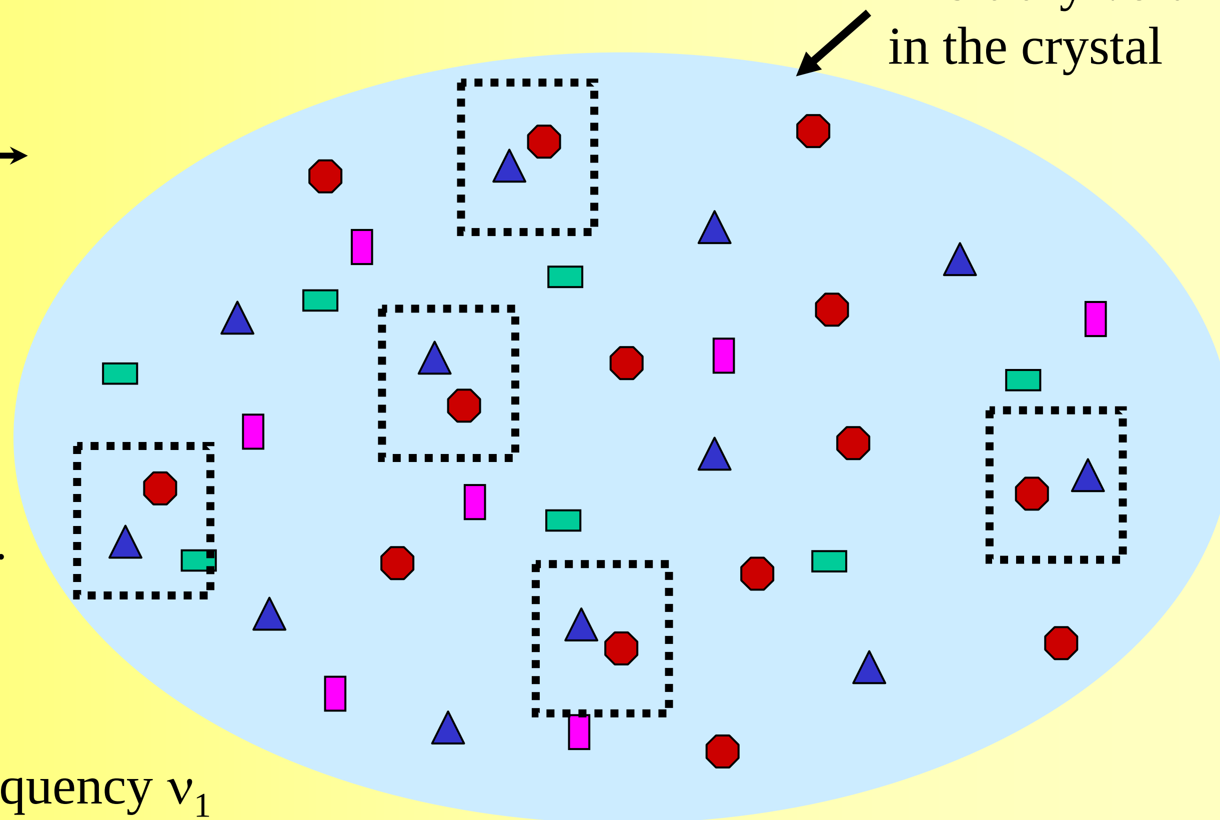
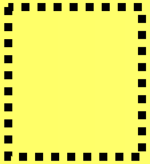
- **Scalability**

Qubit distillation







Ions interacting strongly enough for mutual control. Potential QC

- ion absorbing at frequency ν_1
- ion absorbing at frequency ν_2

Dipole-dipole interaction

$\delta\nu$ – frequency shift due to the dipole-dipole interaction

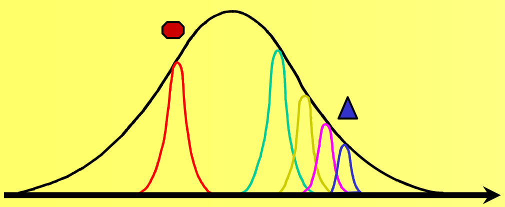
$$\delta\nu \propto \frac{(\Delta\mu)^2}{r^3}$$

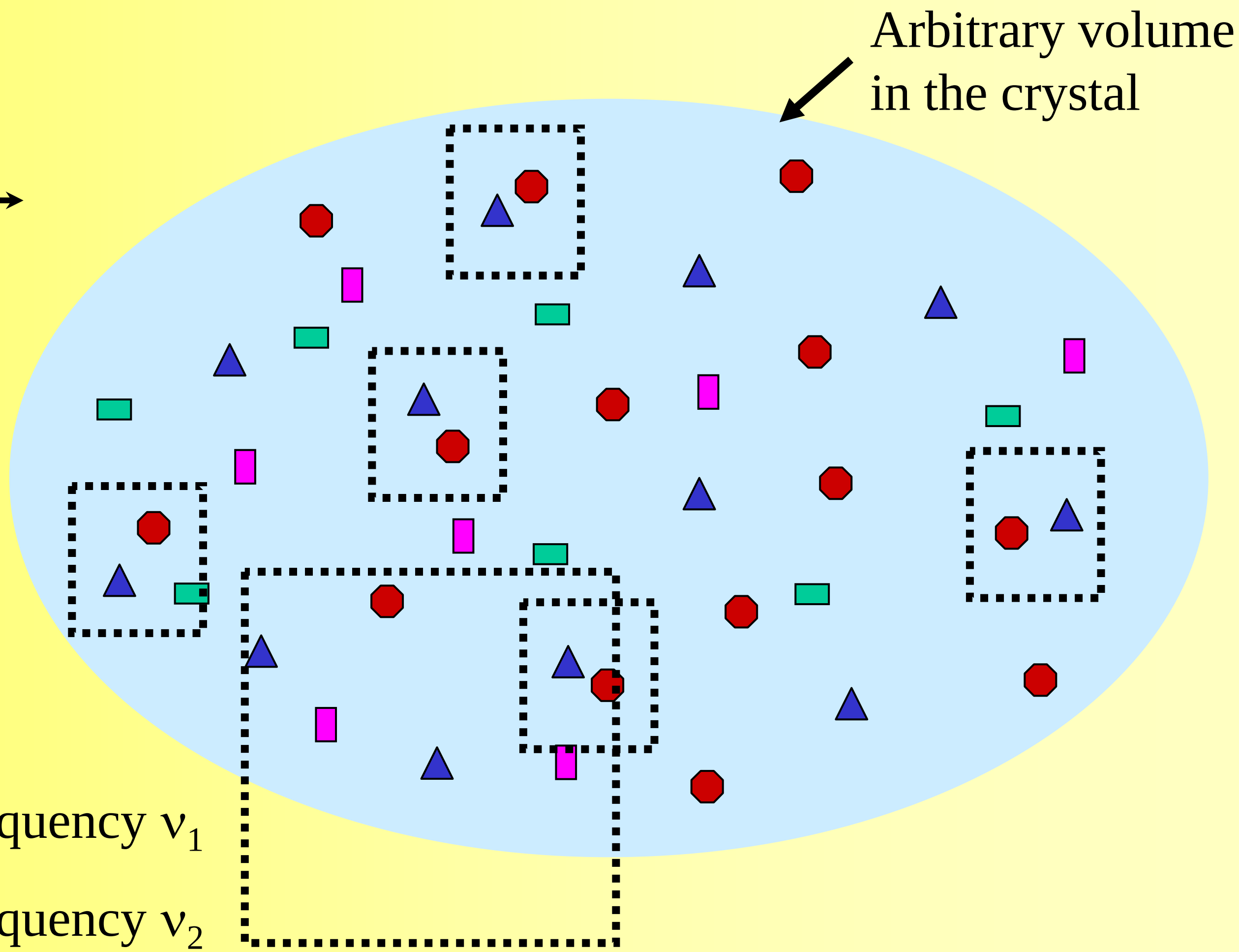
$\Delta\mu$ - difference in dipole moment between ground and excited state
r – distance between interacting ions

Fraction of controllable ions in a qubit (η) scale as $\eta \propto N(\Delta\mu)^2$
N = dopant concentration

Higher dopant concentration, or larger difference in dipole moments between ground and excited state would lead to increased probability to find ions that interact

Scaling

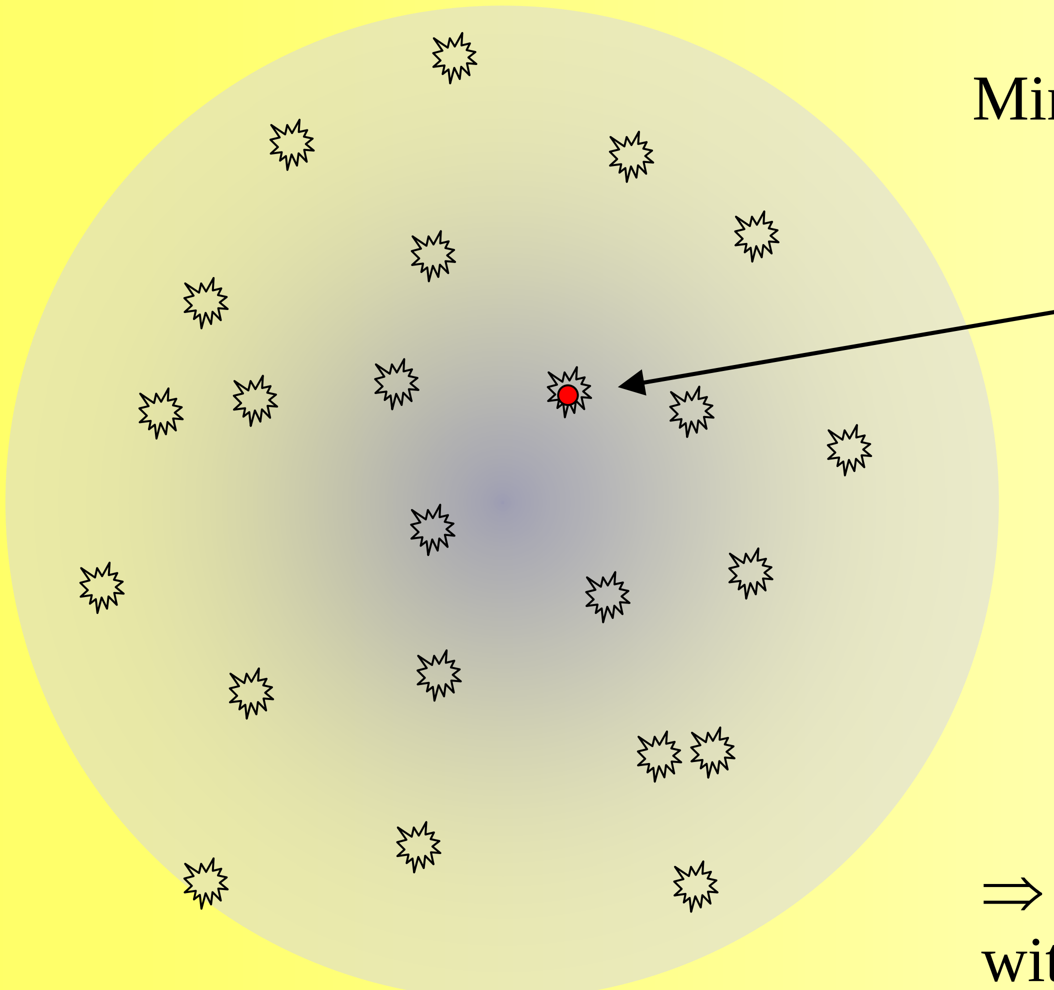




Ions interacting strongly enough for mutual control. Potential QC

- ion absorbing at frequency ν_1
- ion absorbing at frequency ν_2

Readout: Single instance with search approach

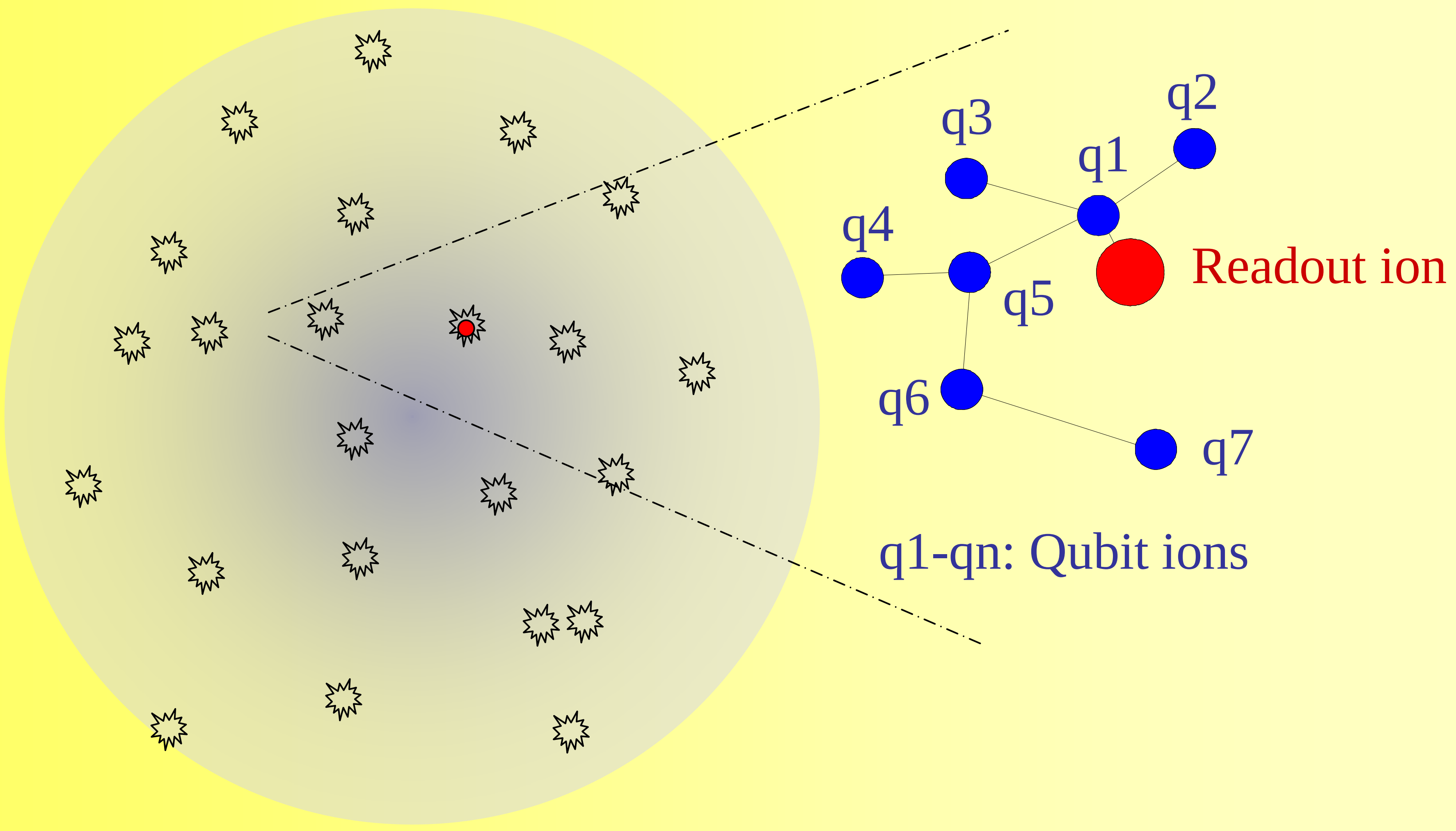


Minimal laser focus

Read out ion of different species

⇒ only read out one with specialised readout ion

Single instance with search approach



Summary
Selected results

- Qubits are initiated in well defined states
- Qubit state-to-state transfer efficiency is 97.5%, simulations predict 98.5%
- Qubit distillation >90%
- Scalable schemes have been developed

Outlook

- Our dye laser system has been frequency stabilised to a few kHz linewidth to carry out high fidelity:
  - Single qubit operations
  - Two-qubit gate operations
- Potential read out ion candidates are investigated

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