Quantum Phase Transitions: Realization and Detection

Michael J. Hartmann

Fernando G.S.L Brandão, Moritz E. Reuter, Martin B. Plenio

Institute for Mathematical Sciences, Imperial College London

QOLS, Blackett Laboratory, Imperial College London

Maria Laach, 16/03/07

Imperial College London

Institute for Mathematical Sciences

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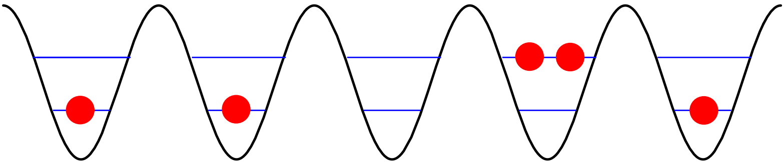
Bose Hubbard Model



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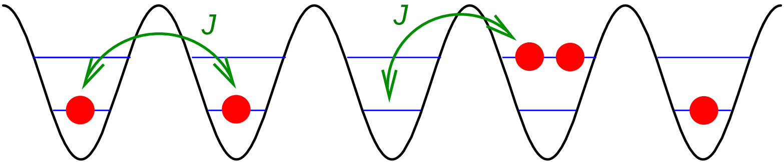

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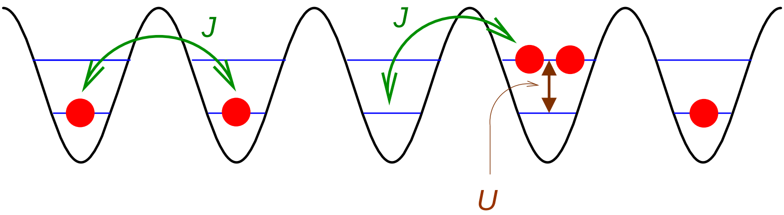

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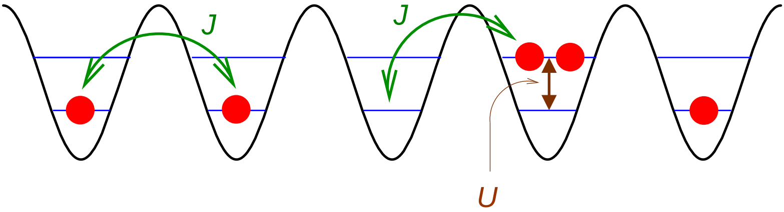

$$H = J \sum_{i=1}^{N} \left(b_i^\dagger b_{i+1} + b_i b_{i+1}^\dagger \right) + U \sum_{i=1}^{N} b_i^\dagger b_i \left(b_i^\dagger b_i - 1 \right)$$

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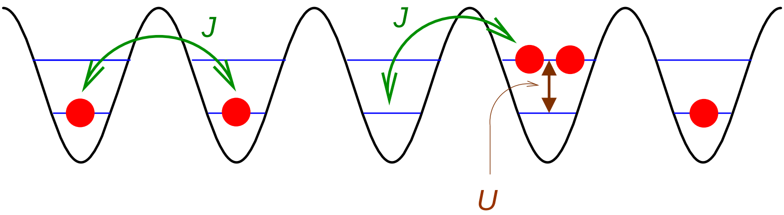

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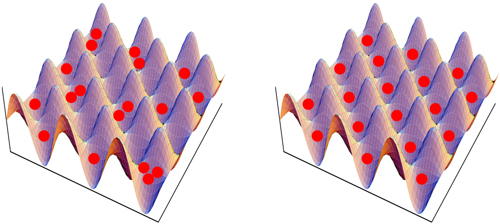
$$\left[b_j, b_l^\dagger \right] = \delta_{j,l}$$

Cold Atoms in Optical Lattices



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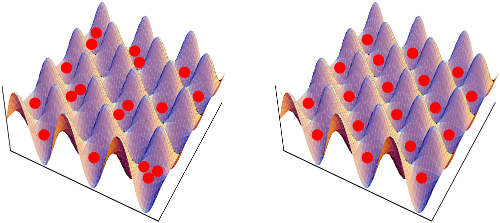

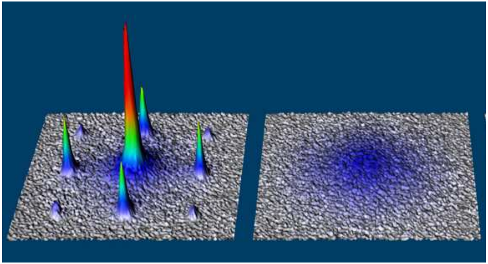

picture: Immanuel Bloch, Mainz

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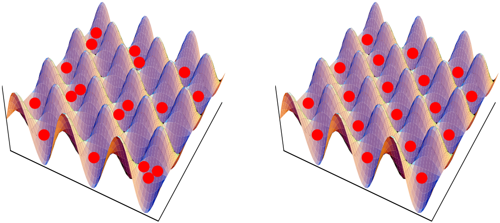
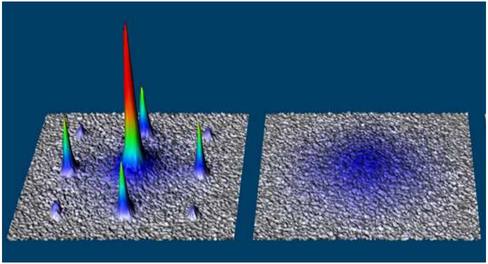


Jaksch et al 1998

Greiner et al 2002



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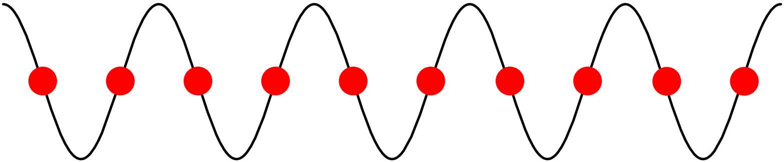
Cold Atoms in Optical Lattices: Limitations



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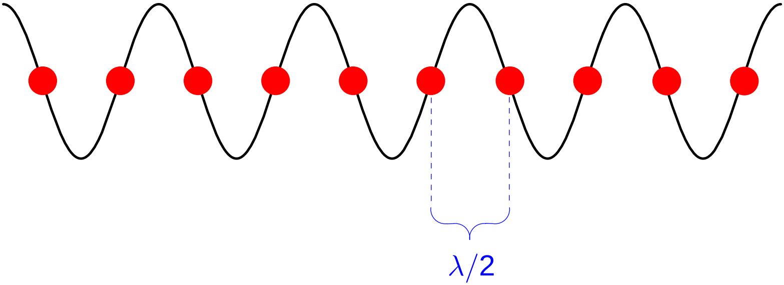

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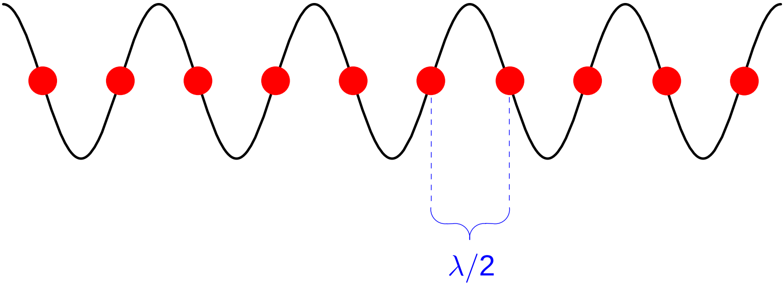

addressing and measuring individual sites is very difficult!

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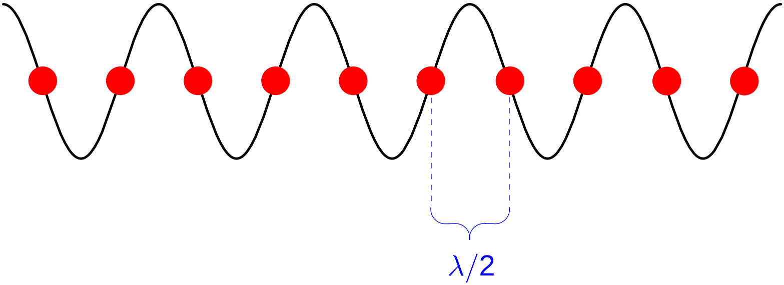
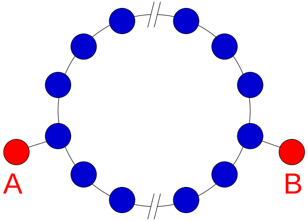


addressing and measuring individual sites is very difficult!

Is there another possible realisation of the Bose Hubbard model that does not suffer from this problem?

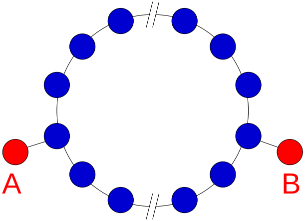
Spectral Gap and Quantum Phase Transition



Ring in the ground state.

Can put excitation in A and look whether it arrives at B.

Spectral Gap and Quantum Phase Transition



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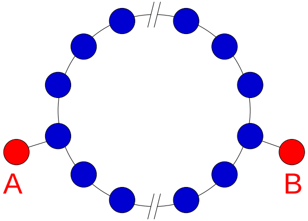
$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \propto e^{-\alpha|x_1-x_2|}$$

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle \propto |x_1-x_2|^n$$

Spectral Gap and Quantum Phase Transition



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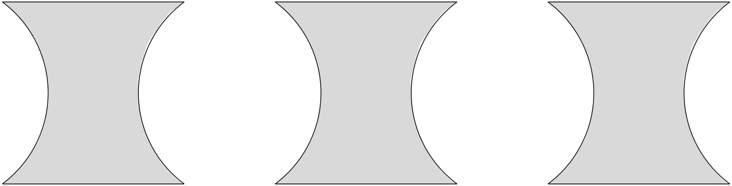
What is the relation to state transfer?

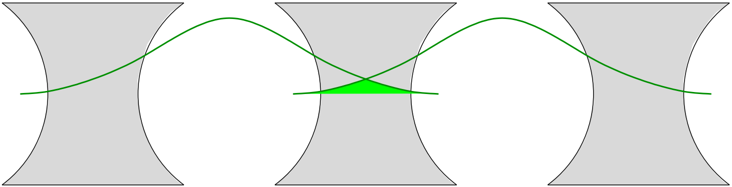
Outline

Outline

The Setup

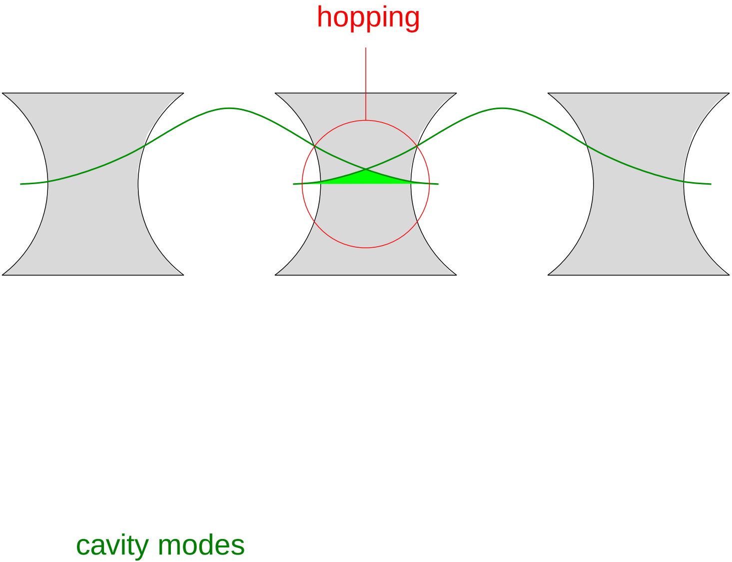


The Setup



cavity modes

The Setup



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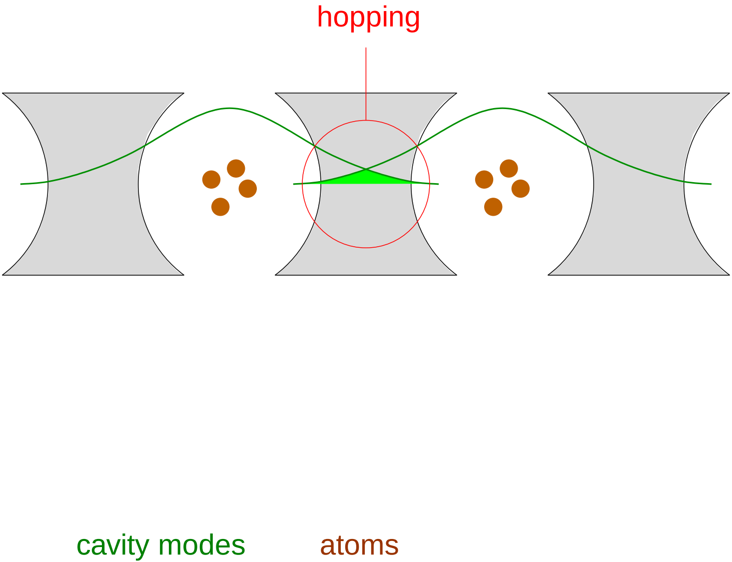

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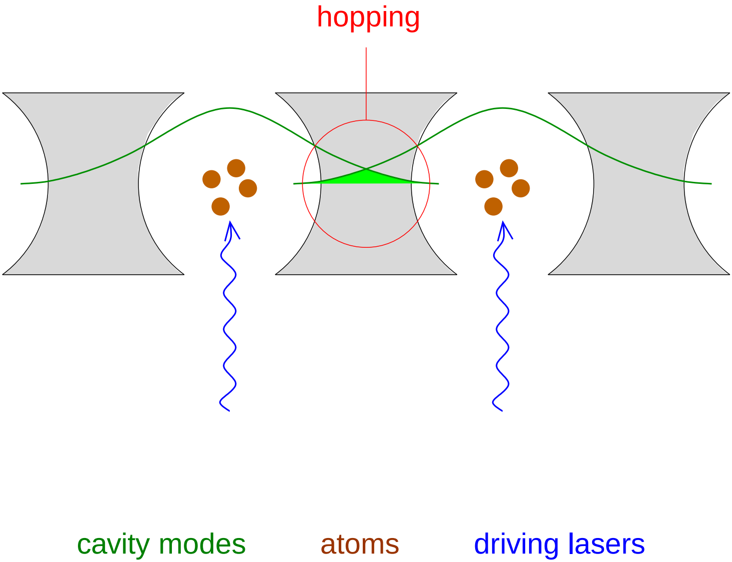

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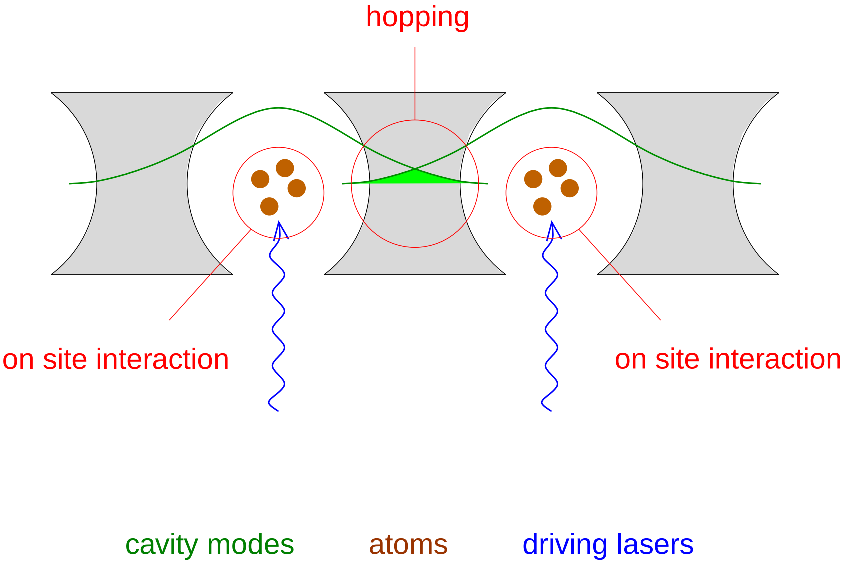
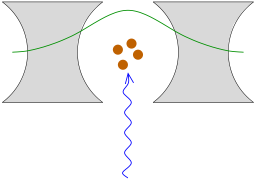

hopping
on site interaction
on site interaction
cavity modes
atoms
driving lasers

The Atoms



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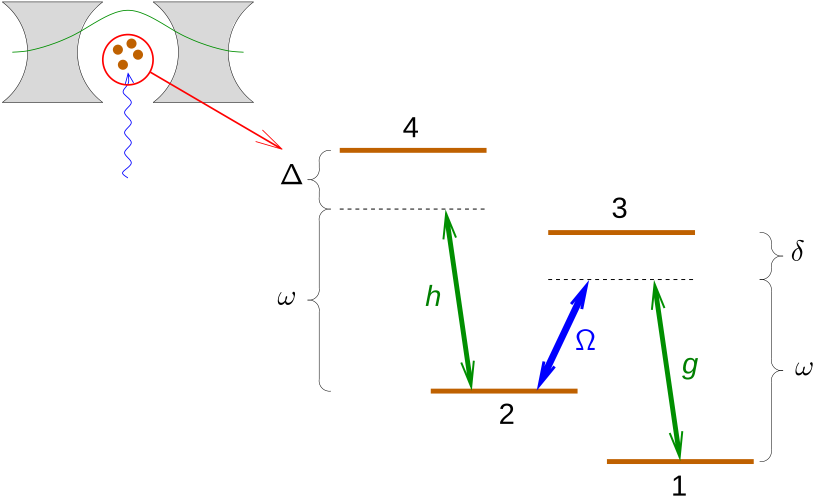

Imamoğlu et al: Phys. Rev. Lett. **79**, 1467 (1997)

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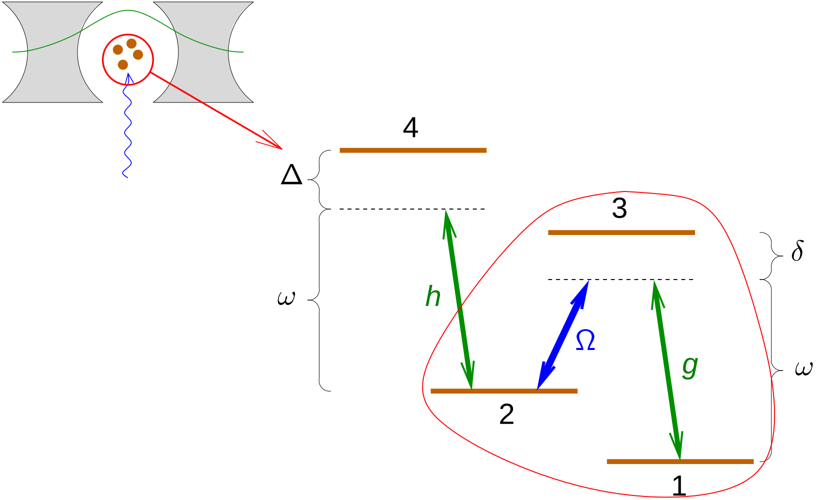

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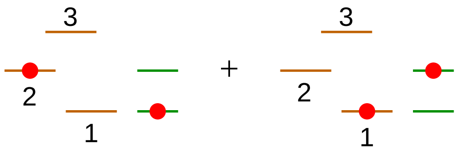
Dark State Polaritons



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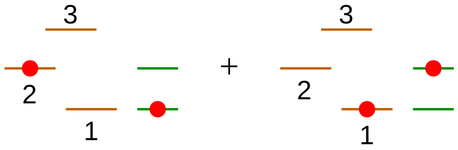
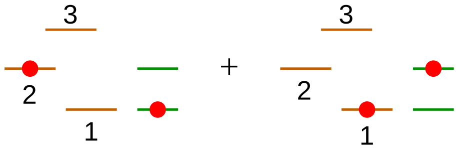


$$p_0^\dagger = \frac{1}{\sqrt{N g^2 + \Omega^2}} \left(\sqrt{N}\, g \, \frac{1}{\sqrt{N}} \sum_{j=1}^{N} |2_j\rangle\langle 1_j| \; - \; \Omega \, a^\dagger \right)$$

Dark State Polaritons



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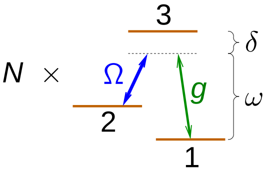
$$\boxed{H = \mu_0\, p_0^\dagger p_0 + \mu_+\, p_+^\dagger p_+ + \mu_-\, p_-^\dagger p_-}$$



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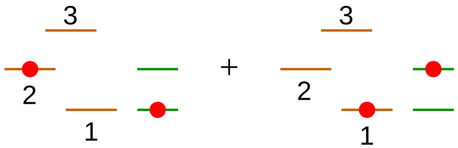

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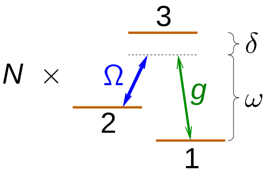
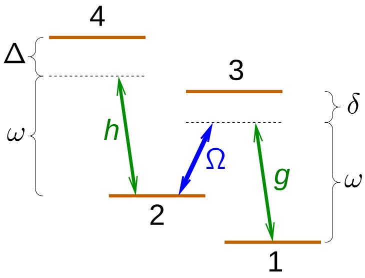


$\Omega \gg \sqrt{N} g$: photonic excitation $\leftrightarrow$ $\Omega \ll \sqrt{N} g$: atomic excitation

$p_0^\dagger$ live only in atomic levels without sponatneous emission

On-Site Interaction and Hopping



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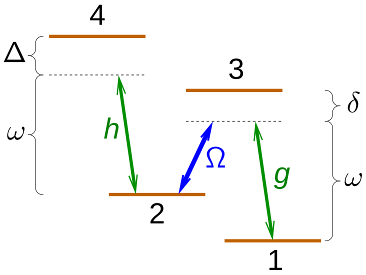
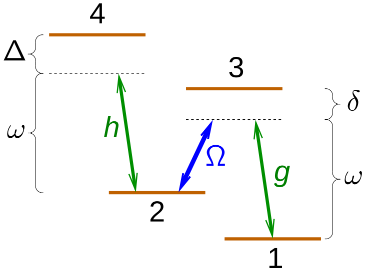


$$|h|\,,\ |\Delta| \ll |\mu_\pm - \mu_0|$$

$\Rightarrow$ only $p_0^\dagger$ couple to level 4.

On-Site Interaction and Hopping



$|h|\,,\ |\Delta| \ll |\mu_{\pm} - \mu_0|$

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$|h| \ll |\Delta| \quad \Rightarrow$ perturbative

$\Rightarrow$ energy shift $\boxed{n\,(n-1)\,U}$
for state with n polaritons

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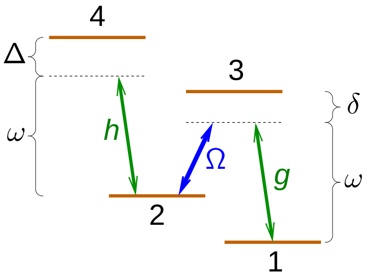

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photon hopping between cavities: $\alpha \sum_j a_j^\dagger a_{j+1} + \text{h.c.}$

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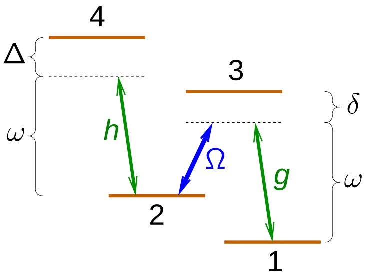

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photon hopping between cavities: $\alpha \sum_j a_j^{\dagger} a_{j+1} + \text{h.c.}$

$$|\alpha| \ll |\mu_{\pm} - \mu_0| \quad \Rightarrow \quad a_j^{\dagger}\, a_{j+1} \propto p_j^{\dagger}\, p_{j+1}$$

Here: p_j means p_0 at site j

Hopping does not introduce coupling between different polariton species because of their different energies

Effective Hamiltonian for Dark State Polaritons

$$H_{\text{eff}} = U \sum_j p_j^\dagger p_j \left(p_j^\dagger p_j - 1\right) + J \sum_j \left(p_j^\dagger p_{j+1} + p_j p_{j+1}^\dagger\right)$$

$$U = -\frac{h^2}{\Delta} \frac{N g^2 \Omega^2}{\left(N g^2 + \Omega^2\right)^2} \qquad J = \frac{\Omega^2}{N g^2 + \Omega^2} 2 \omega \alpha$$

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$$\Delta < 0 \quad \Leftrightarrow \quad U > 0$$

$$\Delta > 0 \quad \Leftrightarrow \quad U < 0$$

new feature: attractive on-site potential

$\rightarrow$ interesting for generation of highly entangled states

Outline

Mott Insulator and Superfluid Phase

$$H_{\text{eff}} = U \sum_j p_j^\dagger p_j \left(p_j^\dagger p_j - 1 \right) + J \sum_j \left(p_j^\dagger p_{j+1} + p_j p_{j+1}^\dagger \right)$$

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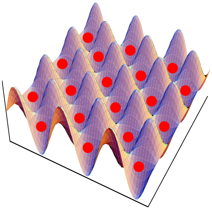
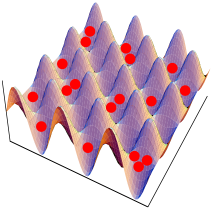


$U \gg J$

hopping suppressed



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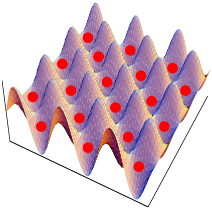

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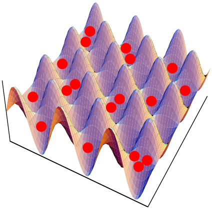

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$$n_l = \langle p_l^\dagger p_l \rangle \qquad F_l = \langle (p_l^\dagger p_l)^2 \rangle - \langle p_l^\dagger p_l \rangle^2$$

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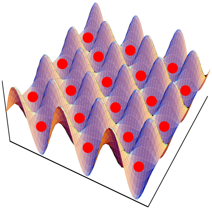

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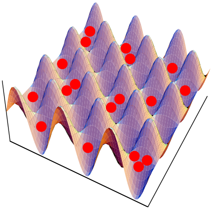

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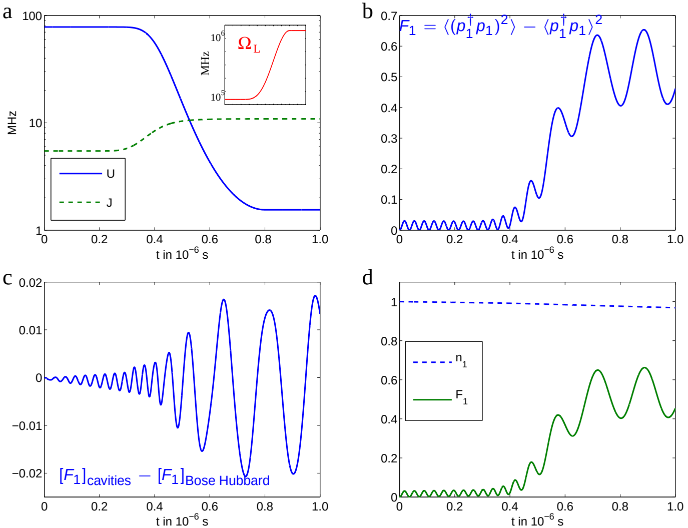
$$\frac{U}{J} = -\frac{h^2}{2\,\omega\,\alpha\,\Delta}\;\frac{N\,g^2}{N\,g^2 + \Omega^2} \;\Rightarrow\; \begin{cases} \Omega \ll \sqrt{N}\,g & \Rightarrow \quad U \gg J \\ \Omega \gg \sqrt{N}\,g & \Rightarrow \quad U \ll J \end{cases}$$

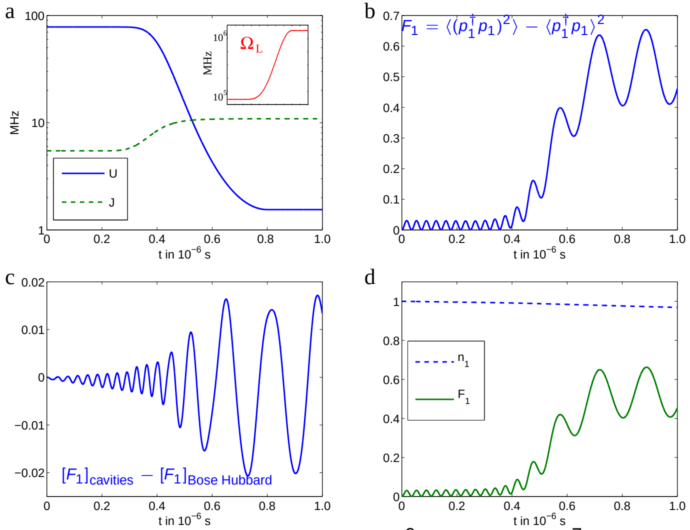
Phase Transition



toroidal micro-cavities: $g \sim h \sim 10^9$Hz, $\Gamma_{SE} \sim 10^7$Hz, $\Gamma_C \sim 10^5$Hz

Spillane et. al., Phys. Rev. A **71**, 013817 (2005)

Phase Transition



can be modifi ed to create photonic Mott insulator "crystalized light"

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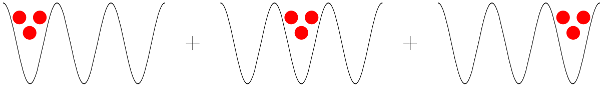
Attractive On-Site Potential

ground state for $U < 0$ and $|U| \gg J$:



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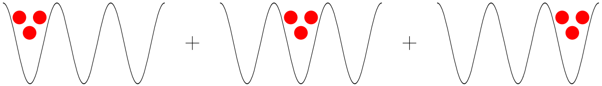
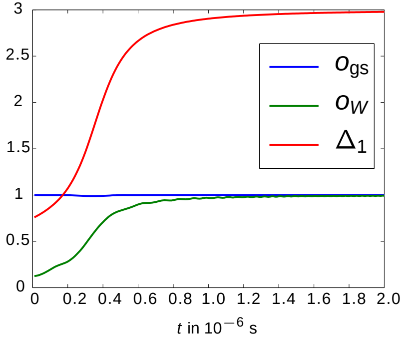
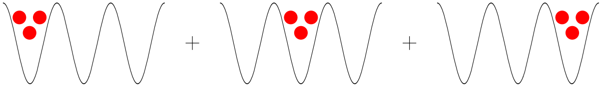


$$J = 10^7 \mathrm{s}^{-1}$$
$$U_{\mathrm{init}} = -2 \times 10^5 \mathrm{s}^{-1}$$
$$U_{\mathrm{fin}} = -4 \times 10^7 \mathrm{s}^{-1}$$
$$o_{\mathrm{gs}} = |\langle \phi(t) | \phi_{\mathrm{gs}}(t) \rangle|$$
$$o_W = |\langle \phi(t) | W_N \rangle|$$
$$F_l = \langle (p_l^\dagger p_l)^2 \rangle - \langle p_l^\dagger p_l \rangle^2$$

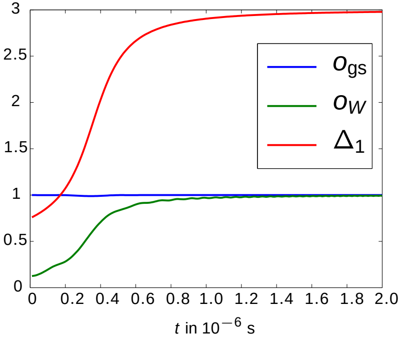


Attractive On-Site Potential

ground state for $U < 0$ and $|U| \gg J$:



$$J = 10^7 \mathrm{s}^{-1}$$
$$U_{\mathrm{init}} = -2 \times 10^5 \mathrm{s}^{-1}$$
$$U_{\mathrm{fin}} = -4 \times 10^7 \mathrm{s}^{-1}$$
$$o_{\mathrm{gs}} = |\langle \phi(t) | \phi_{\mathrm{gs}}(t) \rangle|$$
$$o_W = |\langle \phi(t) | W_N \rangle|$$
$$F_l = \langle (p_l^\dagger p_l)^2 \rangle - \langle p_l^\dagger p_l \rangle^2$$

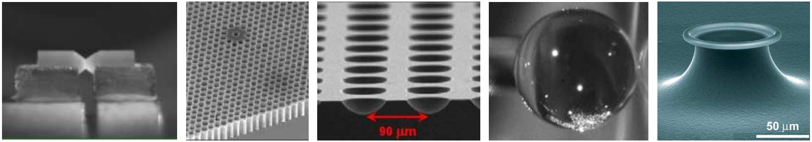


ground state strongly entangled, $F_l \to N - 1$

Outline

Experimental Requirements

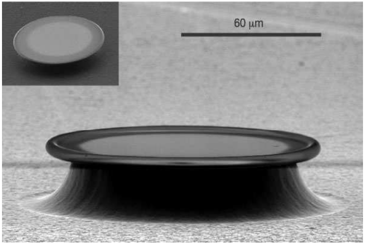
Need $h > 10\,\Gamma_C$ and long-lasting atom trapping

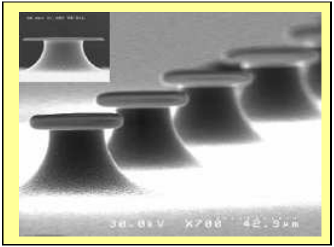


	g/Γ_C achieved	g/Γ_C predicted	atom trapping
Fabry-Perot:	~ 10	~ 40	☺
Photonic band-gap:	~ 5	~ 170	☺
Micro-cavities @ Imperial:	~ 1	?	☺
Micro-sphere:	~ 10	~ 6000	☹
Micro-toroid:	~ 2	~ 60000	☹

Spillane et. al., Phys. Rev. A **71**, 013817 (2005)

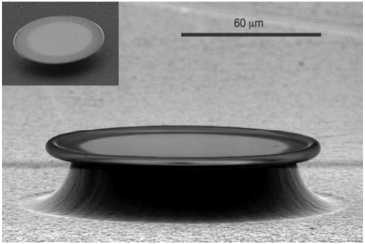
Toroidal Micro-Cavities

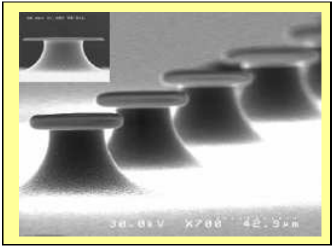




picture: K. Vahala, Caltech

Toroidal Micro-Cavities





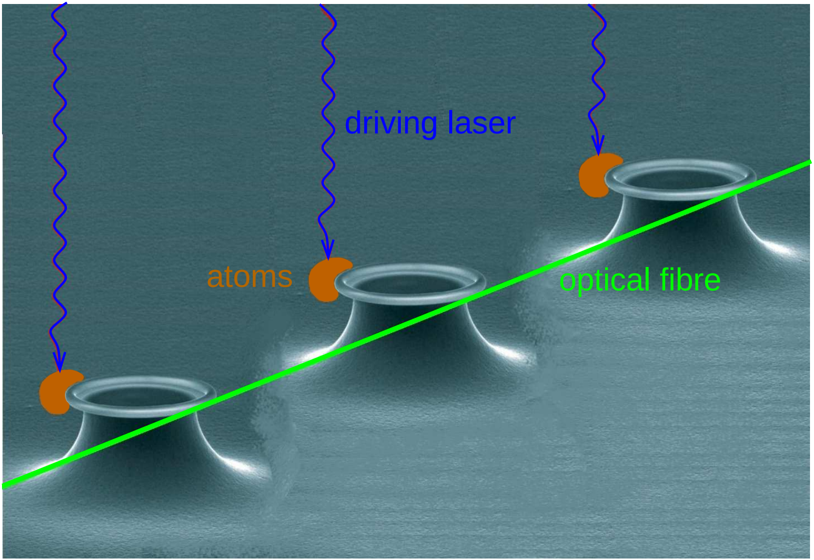
picture: K. Vahala, Caltech

Armani et al: Nature **421**, 925 (2003)

Yang et al: App. Phys. Lett. **83**, 825 (2003)

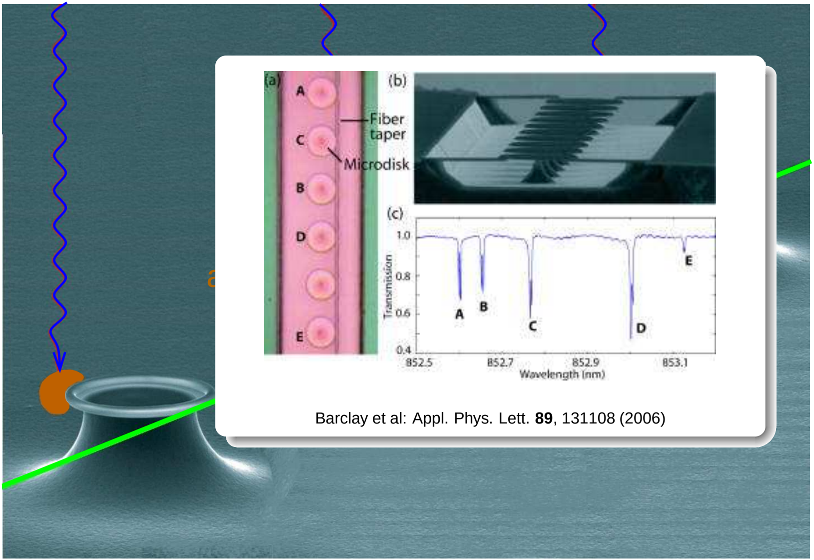
Aoki et al: Nature **443**, 671 (2006)

Array of Toroidal Micro-Cavities



uses picture by T. Kippenberg, MPQ

Array of Toroidal Micro-Cavities

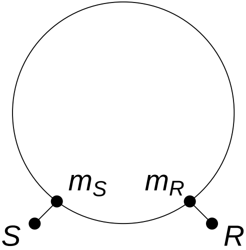


Barclay et al: Appl. Phys. Lett. **89**, 131108 (2006)

uses picture by T. Kippenberg, MPQ

Outline

Excitation and State Transfer in Spin Chains

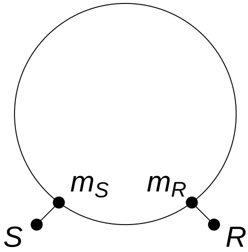


$$H = H_{\text{chain}} + H_{\text{anc}} + H_I$$

$$H_{\text{chain}} = \sum_{i=1}^{N} H_{i,i+1}$$

$$|H_I| \ll |H_{\text{chain}}|,\ |H_{\text{anc}}|$$

Excitation and State Transfer in Spin Chains



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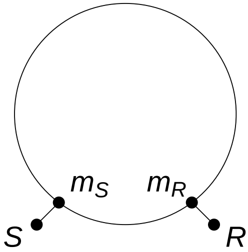
$$H = H_{\text{chain}} + B_a\left(\sigma_S^z + \sigma_R^z\right) + J_a\left(\sigma_S^x \sigma_{m_S}^x + \sigma_R^x \sigma_{m_R}^x\right)$$

Excitation and State Transfer in Spin Chains



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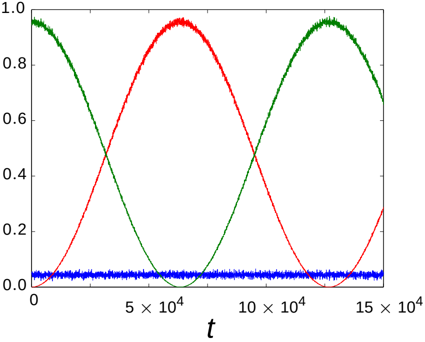
Initial state: $|\Psi_0\rangle = |1_S\rangle \otimes |0_R\rangle \otimes |0_{\text{chain}}\rangle$

Example: $H_{\text{chain}} = \sum_{i=1}^{N} B\,\sigma_i^z + J_x\,\sigma_i^x\,\sigma_{i+1}^x$

has a Quantum Critical Point at $J_x = B$

Transverse Ising Model

$P(0_S, 0_R)$ $P(1_S, 0_R)$ $P(0_S, 1_R)$



$$B = 1,$$

$$J_x = 0.3, J_y = J_z = 0$$

$$B_a = 0.64, J_a = 0.05$$

$$N = 100, m_S = 45, m_R = 55$$

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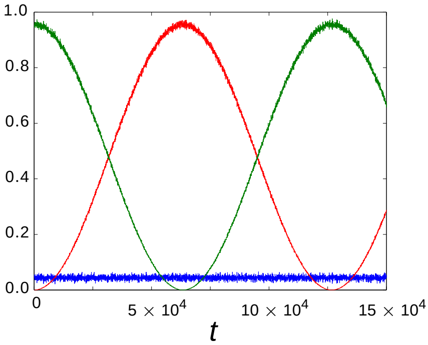

$B = 1,$
$J_x = 0.3, J_y = J_z = 0$
$B_a = 0.64, J_a = 0.05$
$N = 100, m_S = 45, m_R = 55$

$B = 1,$
$J_x = 0.3, J_y = J_z = 0$
$B_a = 0.8, J_a = 0.05$
$N = 600, m_S = 295, m_R = 305$

The Role of Energy Conservation

All $\langle H^n \rangle = const$

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Here: $\langle H \rangle = B_a$ and $\langle H^2 \rangle - \langle H \rangle^2 = J_a^2$

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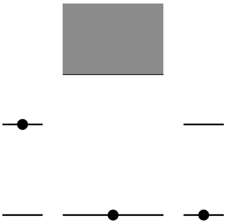

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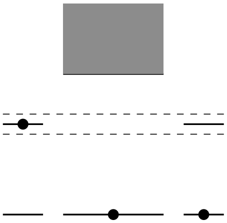

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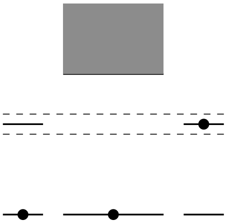

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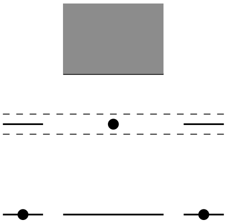

no state available

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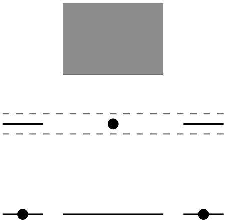
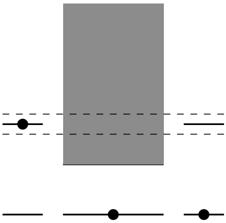
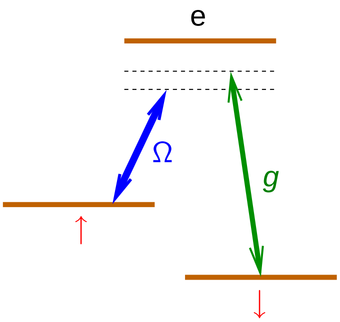


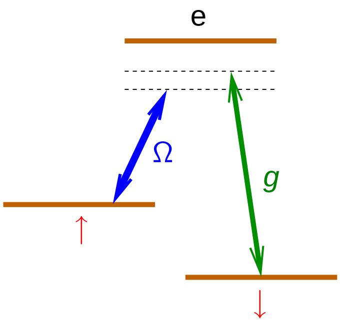
no state available



many states $\Rightarrow$ damping

Current Work: Effective Spin Models





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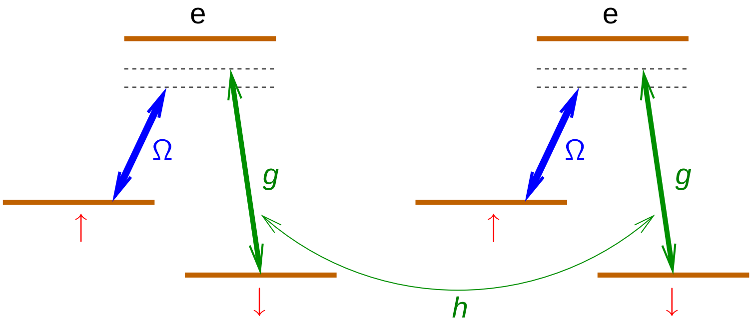

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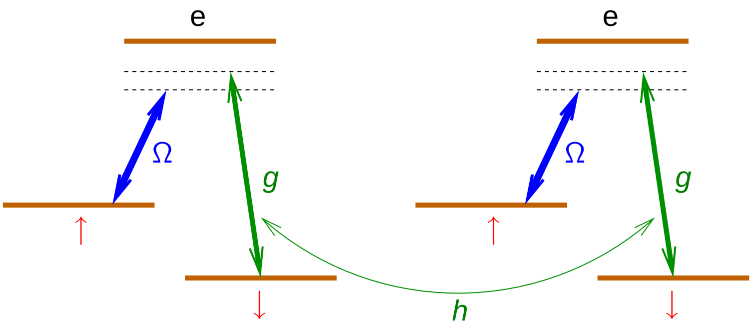

"no" occupation in "e" $\Rightarrow$ "no" spontaneous emission

only virtual photons $\Rightarrow$ "no" cavity decay

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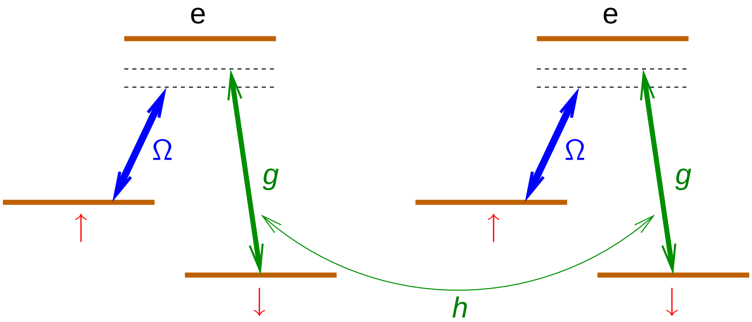


"no" occupation in "e" $\Rightarrow$ "no" spontaneous emission

only virtual photons $\Rightarrow$ "no" cavity decay

$\Gamma_C \ll h$ and $\Gamma_{SE} \ll g$

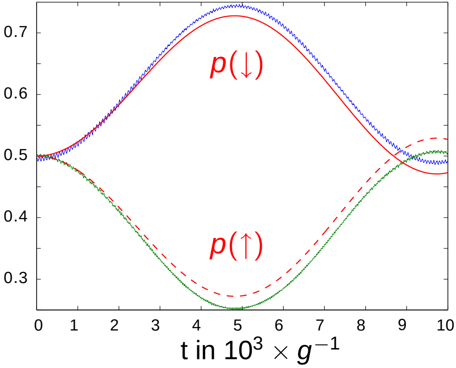
Current Work: Effective Spin Models

$$H_{\text{eff}} = \sum_{j=1}^{N} \left[B\sigma_j^z + J_x \sigma_j^x \sigma_{j+1}^x + J_y \sigma_j^y \sigma_{j+1}^y \right]$$

$B = 0.9 \times 10^{-4} g$

$J_x = 1.3 \times 10^{-4} g$

$J_y = 0.2 \times 10^{-4} g$



Conclusions

1. Realization of a Bose-Hubbard Hamiltonian with polaritons in an array of interacting cavities.
   - experimentally feasible
   - addressing and measuring single sites is possible
   - can realize inhomogeneous and attractive models
   - generalizes to photonic regime $\Rightarrow$ photonic Mott insulator
2. Effective spin models in coupled cavities
3. Excitation and entanglement transfer and spectral gap
   - large gap $\longrightarrow$ good but slow transfer $\longrightarrow$ good channel
   - small gap $\longrightarrow$ fast but imperfect transfer
   - transfer can probe size of gap

References

- Polariton Bose Hubbard model:

  Hartmann, Brandão, Plenio:

  Nature Physics **2**, 849 (2006), quant-ph/0606097

  subsequent proposals:

  Angelakis, Santos, Bose: quant-ph/0606159

  Greentree, Tahan, Cole, Hollenberg:

  Nature Physics **2**, 856 (2006), cond-mat/0609050

- Excitation and entanglement transfer:

  Hartmann, Reuter, Plenio: New J. Phys.: **8**, 1 (2006)

- Effective spin Hamiltonians:

  Hartmann, Brandão, Plenio: coming soon

Thank you very much for listening!

Fernando G.S.L. Brandão

Moritz E. Reuter

Martin B. Plenio

www.imperial.ac.uk/quantuminformation