Wave to Particle dualism
Black body radiation

\[ E(\lambda, T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} \]
Planck's law states that

\[ B_\nu (\nu, T) = \frac{2\nu^3}{c^2} \frac{1}{e^{\frac{\nu}{kT}} - 1} \]

Radiation is quantized

Energy quantum of photon

\[ E = h\nu \]

\[ h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}. \]
Photoelectric effect

Photons of frequency $f$ is shining on target and ejects photo electrons

Stopping potential $V_{\text{stop}}$ measures work function, $K$, of photoelectrons

$$K_{\text{max}} = eV_{\text{stop}},$$

$K$ does not depend on intensity of light but from its energy

$$hf = K_{\text{max}} + \Phi \quad \text{(photoelectric equation)}.$$

$\Phi$ kinetic energy of photo electrons

Energy of bound electron
Measuring $\Phi$

\[ V_{\text{stop}} = \left(\frac{h}{e}\right)f - \frac{\Phi}{e}. \]

\[ \frac{h}{e} = \frac{ab}{bc} = \frac{2.35 \text{ V} - 0.72 \text{ V}}{(11.2 \times 10^{14} - 7.2 \times 10^{14}) \text{ Hz}} = 4.1 \times 10^{-15} \text{ V} \cdot \text{s}. \]

\[ h = (4.1 \times 10^{-15} \text{ V} \cdot \text{s})(1.6 \times 10^{-19} \text{ C}) = 6.6 \times 10^{-34} \text{ J} \cdot \text{s}, \]

Electrons can escape only if the light frequency exceeds a certain value. The escaping electron's kinetic energy is greater for a greater light frequency.

Hint for the „particle“ nature of light

Stopping potential $V_{\text{stop}}$ as a function of the frequency $f$ of the incident light for a sodium target $T$ in the apparatus of Fig. 38-1. (Data reported by R. A. Millikan in 1916.)
Compton effect

\[ E = h\nu = \frac{hc}{\lambda} \rightarrow p = m\,c = \frac{E}{c} = \frac{h}{\lambda} \]

Wave length shift increases with increasing scattering angle
Possible scenarios of photon-electron scattering

(a) An x ray heads toward a target electron.

(b) The x ray can bypass the electron at scattering angle $\phi = 0$.

(c) Or it can scatter at some intermediate angle $\phi$.

(d) Intermediate energy is transferred.

(e) Or it can backscatter at the maximum angle $\phi = 180^\circ$.

(f) Maximum energy is transferred.
\[ E_{\text{kin}} = h\nu - hv^l \]

Conservation of momentum

\[ mv = \frac{h\nu}{c}\sin \frac{\theta}{2} + \frac{hv^l}{c}\sin \frac{\theta}{2} \]

\[ \nu \approx \nu' \]

\[ \frac{1}{2}mv = \frac{h\nu}{c}\sin \frac{\theta}{2} \]

Conservation of energy

\[ \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{4h^2\nu^2 \sin^2 \frac{\theta}{2}}{2mc^2} = h\nu - hv^l \]

* \(1/(hv^2)\) and using \(\nu' \approx \nu\)

\[ \frac{2h}{mc^2} \sin^2 \frac{\theta}{2} = \frac{\nu - \nu^l}{\nu^2} \approx \frac{1}{\nu} - \frac{1}{\nu^l} \]

\[ \lambda = \frac{c}{\nu} \]

\[ \lambda - \lambda^l = \frac{2h}{mc} \sin^2 \frac{\theta}{2} \]

Proof of the „particle“ character of light

Compton wavelength

\[ \lambda_c = \frac{2h}{mc} = 0.0243 \text{Å} \]

\[ \Delta \lambda = \frac{h}{mc} (1 - \cos \theta) \]
Light as a probability wave

Interference is demonstrated using a single photon emission

Measure by photon counter
Neutron scattering

déBroglie relation

\[ E = h\nu = \frac{hc}{\lambda} \rightarrow p = m\nu = \frac{E}{c} = \frac{h}{\lambda} \]

\[ \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m_nE}} \]

\[ \lambda = 2d \sin \theta \]

Proof of „wave“ nature of particles

https://www.google.com/search?client=firefox-b-ab&q=Neutron+powder+diffraction
Properties of the neutron

- Mass
  - Neutron \( m_n = 1.675 \times 10^{-24} \text{g} \)
  - Electron \( m_e = 9.10 \times 10^{-28} \text{g} \)
  - Photon 0

- Dispersion
  \[
  E = \frac{\hbar^2 k^2}{2m} = 2.07214 \text{meV} \AA^2 \times k^2
  \]
  \[
  \lambda = \frac{2\pi}{k} = \frac{9.044605 \text{Å}}{\sqrt{E[\text{meV}]}}
  \]
  \[
  \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2m_nE}}
  \]

- Energy at \( \lambda = 5 \text{Å} \)
  - Neutron \( E = 3.3 \text{ meV} \sim 40 \text{K} \)
  - Electron \( E \sim 6 \text{ eV} \sim 70,000 \text{ K} \)
  - Photon \( E = \hbar c/\lambda \sim 2.5 \text{ keV} \sim 3 \text{ MK} \)

- Neutron energy and wavelengths are **perfectly matched** to condensed matter physics.

Prof. A. Zheludev PSI- Switzerland
Fission

- Neutron-induced nuclear decay

- Average yield:
  - thermal fission of $^{235}\text{U}$: 2.5 n
  - fast fission of $^{238}\text{U}$: 2.6 n
  - spontaneous decay of $^{238}\text{U}$: 2.4 n

- $^{235}\text{U}$ example:

  $^{235}\text{U} + n_{th} \rightarrow (^{236}\text{U})^* \rightarrow (^{139}\text{I})^* + (^{96}\text{Y})^* - n$

  $(^{139}\text{I})^* \rightarrow (^{138}\text{I})^* - n \rightarrow (^{138}\text{Xe})^* + \beta^- \rightarrow (^{138}\text{Cs})^* + \beta^- \rightarrow (^{138}\text{Ba}) + \beta^-$

  $(^{96}\text{Y})^* \rightarrow (^{95}\text{Y})^* - n \rightarrow (^{95}\text{Zr})^* + \beta^- \rightarrow (^{95}\text{Nb})^* + \beta^- \rightarrow (^{95}\text{Mo}) + \beta^-$

- Moderation will sustain chain reaction
Neutron Moderation

Maxwellian Distribution

\[ \Phi \sim v^3 e^{-mv^2/2k_BT} \]

“Fast” neutrons: \( v = 20,000 \text{ km/sec} \)

Liquid Hydrogen

Heavy Water (\( \text{D}_2\text{O} \))

Hot Graphite

Neutron velocity \( v \) (km/sec)
Reactors

- Control rods: Boron
- Fuel rods: Uranium
- Beam tube
- Moderator + collant
  - $D_2O$ for $U^{238}$ fuel
  - $H_2O$ for $U^{235}$-enriched fuel
- Bio shield: concrete + steel

![FRM-II (Munich)](image)

![ILL (Grenoble)](image)

![Graph showing thermal neutron flux](image)
Reactor neutrons

- Spectrum

\[ n_v \propto v^2 \exp\left(-\frac{mv^2}{2\kappa T}\right) \]

\[ E_{\text{max}} = \kappa T \]

\[ \bar{E} = \frac{\kappa T}{2} \]

Moderator temperature

- Cold source: \( E/\kappa \sim 20\text{K} \)

Beam tube

- Hot source: \( E/\kappa \sim 2000\text{K} \)

ILL

Cryogen circulation

Liquid hydrogen

Thermal (equilibrium with moderator)

Fast (never scattered)

Epithermal (scattered a few times)

Neutron energy (eV)

Flux (arb. u.)
Electron diffraction

\[ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e E}} \]

Wave length for e-beam (120eV)

\[ \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{5.91 \times 10^{-24} \text{ kg} \cdot \text{m/s}} = 112 \text{ pm.} \]
e-beam vs light diffraction

Fig. 38-9  (a) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of the incident beam. Photographs of the diffraction patterns when the incident beam is (b) an x-ray beam (light wave) and (c) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other. (Photo (b) Cameca, Inc. Photo (c) from PSSC film “Matter Waves,” courtesy Education Development Center, Newton, Massachusetts)
Physics 1 for Nanoscience & Nanotechnology

Level of knowledge  4

1. Explain the refraction law in terms of wave length and velocity?

2. What means „coherence“? How one can quantify?

3. Give an expression for intensity distribution of single slit diffraction. Express the phase term for constructive interference.

4. Express Bragg equation for x-ray diffraction, which meaning has the value „d“?

5. Which experiments proof the particle character of a wave? Which experiments proof the wave character of a particle?
Physics 1 for Nanoscience & Nanotechnology

Level of knowledge  4

1. Explain the refraction law in terms of wave length and velocity ?
   \[ v = \frac{c}{n}, \quad \lambda = \frac{\lambda}{n} \]

2. What means „coherence“? How one can quantify?
   capability to form interference, spatial & temporal coherence length

3. Give an expression for intensity distribution of single slit diffraction. Express the phase term for constructive interference.
   \[ I(a) = \sin^2(a)/a^2 \text{ using } a = \frac{\pi}{\lambda} \ a \sin \theta \text{, max at } a = \frac{1}{2} (2n+1) \lambda \]

4. Express Bragg equation for x-ray diffraction, which meaning has the value „d“?
   \[ \lambda = 2d \sin \theta, \text{ d is the lattice spacing between atomic planes} \]

5. Which experiments proof the particle character of a wave? Which experiments proof the wave character of a particle?
   Photoeffect + Compton effect; electron and neutron diffraction
Beating of frequency

\[ E_1 + E_2 = E_0 \left[ \sin 2\pi(v_1 t - \frac{x}{\lambda_1}) + \sin 2\pi(v_2 t - \frac{x}{\lambda_2}) \right] \]

\[ E_1 + E_2 = 2E_0 \sin 2\pi \left[ \frac{(v_1 + v_2)}{2} t - \frac{1}{2}\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)x \right] \cos 2\pi \left[ \frac{(v_1 - v_2)}{2} t - \frac{1}{2}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)x \right] \]
Shape of a wave group

\[ \Delta x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} = \frac{\frac{h}{mv_1} \cdot \frac{h}{mv_2}}{\frac{h}{mv_2} - \frac{h}{mv_1}} \approx \frac{\frac{h^2}{m^2 v^2}}{\frac{h}{v_1 v_2}} \approx \frac{h}{\Delta (mv)} = \frac{h}{\Delta p} \]

\[ \Lambda = \frac{2\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} \approx \frac{2\lambda^2}{\Delta \lambda} \]

The smaller \( \Delta x \) the smaller \( \Delta \lambda \)

Momentum and position can not be determined precisely simultaneously

\[ \Delta x \cdot \Delta p_x \geq \hbar \]
\[ \Delta y \cdot \Delta p_y \geq \hbar \quad \text{(Heisenberg’s uncertainty principle).} \]
\[ \Delta z \cdot \Delta p_z \geq \hbar \]
Heisenberg’s uncertainty principle

\[ \Delta x = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} = \frac{\frac{h}{mv_1} \frac{h}{mv_2}}{\frac{h}{mv_2} - \frac{h}{mv_1}} \approx \frac{\frac{h^2}{m^2v^2}}{\frac{h}{v_1v_2} - \frac{h}{v_1v_2}} \approx \frac{h}{\Delta (mv)} = \frac{h}{\Delta p} \]

\[ \Delta x \cdot \Delta p_x \geq \hbar \]
\[ \Delta y \cdot \Delta p_y \geq \hbar \] (Heisenberg’s uncertainty principle).
\[ \Delta z \cdot \Delta p_z \geq \hbar \]

Proof for diffraction

1. minimum momentum

\[ \sin \theta \approx \theta = \frac{\lambda}{\Delta x} \]
\[ \theta \approx \frac{\Delta v_x}{v_0} \]

\[ \Delta v_x \cdot \Delta x = \lambda \cdot v_0 = \frac{h}{mv_0} \cdot v_0 \]

\[ m \Delta v_x \cdot \Delta x = \Delta p \cdot \Delta x = h \]
Schrödinger equation

Light and small particles can be described by a matter wave = probability wave

\[ \Psi(x, y, z, t) = \psi(x, y, z) e^{-i\omega t}, \]

Probability that a particle will be found at specific position \( x \) (e.g. at detector) in a specific time interval is proportional to \( |\psi|^2 \), whereas \( \psi \) is often complex, \( |\psi|^2 \) is always real = probability density, how to find \( \psi \)? It must satisfy

\[ \frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{\hbar^2} [E - U(x)]\psi = 0 \]

(Schrödinger’s equation, one-dimensional motion),

\( E \) is total energy of the particle, \( U(x) \) is potential energy,

→ Schrödinger equation is a basic principle of physics, it cannot be derived !!!

In case of \( U=0 \) → free particle function, \( E=E_{\text{kin}} \)

\[ \frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{\hbar^2} \left( \frac{mv^2}{2} \right) \psi = 0, \quad \frac{d^2\psi}{dx^2} + k^2\psi = 0 \]

\[ \psi(x) = Ae^{ikx} + Be^{-ikx}, \]
Probability density for a particle moving along x direction is constant for all x.
Barrier tunneling

Find wave function $\psi(x)$ for this problem

Incoming wave, mainly reflected

Transmitting wave decays exponentially

Transmission coefficient $T$

$$T = e^{-2bl},$$
$$b = \sqrt{\frac{8\pi^2 m(U_b - E)}{\hbar^2}}.$$
Scanning tunnel microscopy

Tunnel current between tip and surface changes exponentially with distance L,
Sensitive feedback value

Polymer surface relief grating

Harmonics

$$\lambda = \frac{2L}{n}, \quad \text{for } n = 1, 2, 3, \ldots$$

$$f = \frac{v}{\lambda} = n \frac{v}{2L}, \quad \text{for } n = 1, 2, 3, \ldots$$

First harmonic

Second harmonic

Third harmonic
**Electron in a infinite potential wall**

\[-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z) + V(x, y, z)\psi(x, y, z) = E\psi(x, y, z)\]

\(x, y \to \infty\), stacking \(\parallel z\)

\[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \psi(z) + V(z)\psi(z) = E\psi(z)\]

Within QW \(V=0\)

\[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \psi(z) = E\psi(z)\]

**Ansatz**

\[\psi(z) = A \sin( kz) + B \cos(kz)\]

**Boundary condition**

\[\psi(0) = \psi(L) = 0 \quad \rightarrow \quad B=0 \quad \psi(z) = A \sin( kz)\]

\[E_n = \frac{\hbar^2 \pi^2 n^2}{2m \times L^2}\]

\[\psi(z) = \sqrt{\frac{2}{L}} \sin \left( \frac{\pi n z}{L} \right)\]
Quantization, infinite potential wall

\[ E_z = E_n \]

\[ E_n = \frac{\hbar^2 \pi^2 n^2}{2m* L^2} \]

Electron energy is quantized

\[ E = E_z + E_{x,y} = E_n + \frac{\hbar^2 |k_{x,y}|^2}{2m*} \]

Harrison book
Tutorial about Physics 1

30.1. Mechanics 9:00

8.2. Electrodynamics 9:00

12.2. Wave optics 9:00

Written exam 20.2.2019

Elegible to write the exam are those students which have submitted examination sheets and have received 50% of possible points. !!!

You have the chance to complete your submission up to 15.2. 2019, Next chance for exam at 20.3. 2019 (2nd exam)