

Solid state physics for Nano



Lecture 9: Magnetism

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Diamagnetism and Paramagnetism

Mater in magnetic field (see basic course electrodynamics)

$$B = \mu_0 H + M$$

$$B = \mu_0 H (1 + \chi_m)$$

$$B = \mu_0 \mu H$$

$$\chi_m = \frac{\mu_0 M}{B}$$

M – magnetisation

χ – magnetic susceptibility

μ - magnetic permeability

Within magnetic field, alignment of permanent magnetic dipoles

→ paramagnetism

Diamagnetism $\chi_m < 0$

All atoms with paired spins : $S=0$



$$\chi_m = \frac{\mu_0 Z e^2 N}{6m} \langle r^2 \rangle$$

$$\chi_m \approx 10^{-6}$$

Langevin equation

N atoms/unit volume

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

Electron distribution within the atom

[Paul Langevin](#)

[1872 - 1946](#)

Explanation:

The Langevin theory of diamagnetism applies to materials containing atoms with closed shells. A magnetic field with strength B , applied to an electron with charge e and mass m , gives rise to [Larmor precession](#) with frequency $\omega = eB / 2m$. The number of revolutions per unit time is $\omega / 2\pi$, so the current for an atom with Z electrons is

$$I = -\left(\frac{Ze^2 B}{4\pi m}\right)$$

The [magnetic moment](#) of a current loop is equal to the current times the area of the loop. Suppose the field is aligned with the z axis. The average loop area can be given as $\pi \langle \rho^2 \rangle$ where $\langle \rho^2 \rangle$ is the mean square distance of the electron perpendicular to the z axis.

The magnetic moment is

$$\mu = -\left(\frac{Ze^2 B}{4m}\right) \langle \rho^2 \rangle$$

If the distribution of charge is spherically symmetric, we can suppose that the distribution of x, y, z coordinates are independent and identically distributed. Then

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle = 1/3 \langle r^2 \rangle$$

where $\langle r^2 \rangle$ is the mean square distance of the electrons from the nucleus. Therefore,

$$\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = 2/3 \langle r^2 \rangle$$

If N is the number of atoms per unit volume, the [diamagnetic susceptibility](#) in SI units is

$$\chi_m = \frac{\mu_0 \mu N}{B} = \frac{\mu_0 Z e^2 N}{6m} \langle r^2 \rangle$$

Paramagnetism

Paramagnetism $\chi_m > 0$

- Atoms with odd number of electrons $S \neq 0$
- Free atoms/ions with partly filled inner shells
- Metals...

Magnetic moment

$$\mu = \gamma \hbar J = -g \mu_B J$$

γ – gyromagnetic ratio = magn. moment/angular momentum

μ_B – Bohr magneton

g – Lande factor = Number of Bohr magnetons / angular momentum in units of $\hbar/2\pi$

For atom

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

$\hbar J$ Is sum of orbital momenta

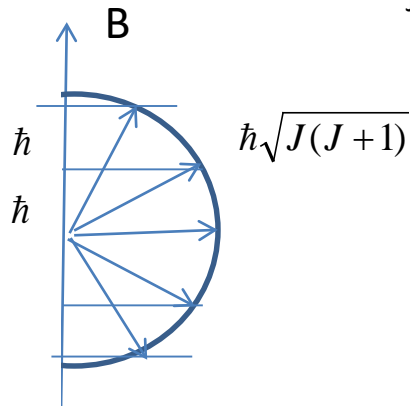
Russels Saunders Coupling

$$\hbar J = \hbar L + \hbar S$$

Energy niveau splitting of B-field

$$\Delta E = m_J g \mu_B B = \mu B$$

$$m_J = J, J-1, J-2, \dots, -J$$



Spin alignment quantized

Spin never parallel B

Energy levels of a system in a magnetic field

In two level system

$$U = \pm \mu B$$

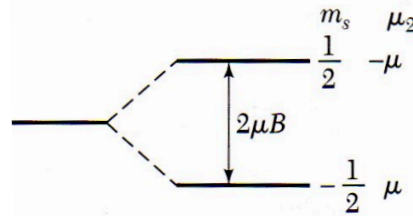


Figure 2 Energy level splitting for one electron

$m_j = J, J-1, J-2, \dots, -J$, for $L=0, S=1/2, m_j = \pm 1/2$

Population of upper and lower level:

$$\frac{N_1}{N} = \frac{\exp(\mu B / kT)}{\exp(\mu B / kT) + \exp(-\mu B / kT)}$$

$$\frac{N_2}{N} = \frac{\exp(-\mu B / kT)}{\exp(\mu B / kT) + \exp(-\mu B / kT)}$$

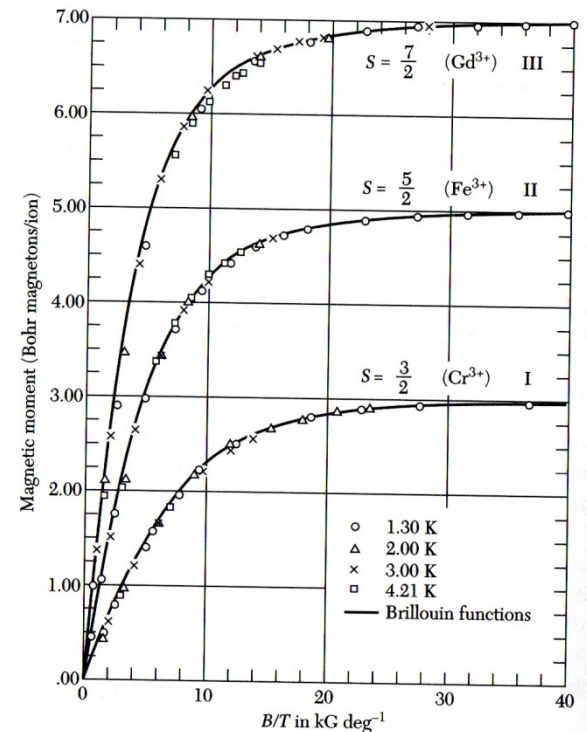
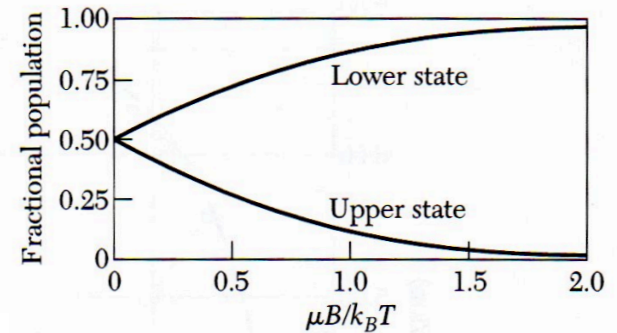
Projection of magnetic moment of upper/lower state is $-\mu$ and $+\mu$, resultant magnetization is $x = \mu B / kT$

$$M = (N_1 - N_2)\mu = N\mu \frac{e^x - e^{-x}}{e^x + e^{-x}} = N\mu \tanh x$$

For $x \ll 1$ $\tanh x = x$

$$M = N\mu(\mu B / kT)$$

$$U = -m_J g \mu_B B = -\mu B$$



In atom with angular momentum quantum number J there are $2J+1$ equally spaced energy levels

$$M = N \frac{\sum_{-J}^J (g\mu_B m_J) \exp(\frac{g\mu_B m_J B}{kT})}{\sum_{-J}^J \exp(\frac{g\mu_B m_J B}{kT})} \rightarrow M = Ng\mu_B JB_J(y) \quad y = \frac{g\mu_B JB}{kT}$$

$$M_{\max} = Ng\mu_B J$$



Leon Brillouin
1889 - 1969

$B_J(y)$ – Brillouin function

$$B_J(y) = \frac{2J+1}{2J} \coth(\frac{2J+1}{2J} y) - \frac{1}{2J} \coth(\frac{y}{2J})$$

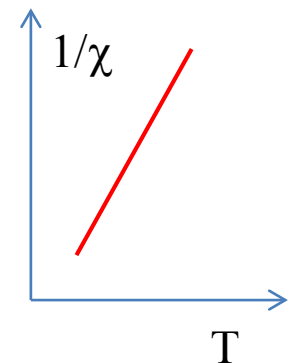
For large $B(y) \rightarrow$ Langevin

For $y \ll 1$: $\coth(y) = 1/y + y/3 - y^3/45 + \dots$

$$B_J(y) \approx \frac{y(J+1)}{3J}$$

$$\chi_m = \frac{\mu_0 M}{B} = \frac{\mu_0 Ng^2 \mu_B^2 J(J+1)}{3kT} = \frac{\mu_0 N \mu_{\text{eff}}^2}{3kT} = \frac{C}{T}$$

Curie - law



Measurement of $\chi(T)$

$\rightarrow \mu_{\text{eff}} = p \mu_B$

Table 1 Effective magneton numbers p for trivalent lanthanide group ions

(Near room temperature)

Ion	Configuration	Basic level	$p(\text{calc}) = g[J(J + 1)]^{1/2}$	$p(\text{exp}),$ approximate
Ce ³⁺	$4f^1 5s^2 p^6$	$^2F_{5/2}$	2.54	2.4
Pr ³⁺	$4f^2 5s^2 p^6$	3H_4	3.58	3.5
Nd ³⁺	$4f^3 5s^2 p^6$	$^4I_{9/2}$	3.62	3.5
Pm ³⁺	$4f^4 5s^2 p^6$	5I_4	2.68	—
Sm ³⁺	$4f^5 5s^2 p^6$	$^6H_{5/2}$	0.84	1.5
Eu ³⁺	$4f^6 5s^2 p^6$	7F_0	0	3.4
Gd ³⁺	$4f^7 5s^2 p^6$	$^8S_{7/2}$	7.94	8.0
Tb ³⁺	$4f^8 5s^2 p^6$	7F_6	9.72	9.5
Dy ³⁺	$4f^9 5s^2 p^6$	$^6H_{15/2}$	10.63	10.6
Ho ³⁺	$4f^{10} 5s^2 p^6$	5I_8	10.60	10.4
Er ³⁺	$4f^{11} 5s^2 p^6$	$^4I_{15/2}$	9.59	9.5
Tm ³⁺	$4f^{12} 5s^2 p^6$	3H_6	7.57	7.3
Yb ³⁺	$4f^{13} 5s^2 p^6$	$^2F_{7/2}$	4.54	4.5

Paramagnetism of ions

Calculation of μ_{eff} from electron configuration of atoms, considering

- Pauli principle
- Hund's rule

Electrons in partially filled shell first towards maximum S $\uparrow \uparrow$ followed by maximizing L

$J = L - S$ for shell below half filling

$J = L + S$ for shell above half filling

Example Ce^{3+} , 1 f-electron, $L=3$; $S=1/2 \rightarrow J=L-S = 5/2$, experimental finding often $L=0$

Orbital moment is „quenched“ ,

caused by time average in non-cubic crystal field (see later)

Example 3d elements do show spin magnetism only !!!! – no orbital magnetism

Solid state magnetism

Considers: (1) interatomic interaction ; (2) interaction of magnetic moments

Metals:

→ magnetism of electrons in conduction band

→ Magnetism of inner, partially filled shells : i.e. Fe 3d group, rare elements 4f shell

Ion crystals:

→ L=0 – spin magnetism, rare earth elements L is not quenched , 4f electrons are screened by 5p, 5d and 6s electrons against external crystal fields

Paramagnetism of conducting electrons

magnetic moment of single electron $\mu_B = \frac{e\hbar}{2mc}$ per electron (μ Bohr)

Expectation of classic free electron gas $M = N\mu_B L\left(\frac{\mu_B B}{kT}\right) \approx n\mu_B \frac{\mu_B B}{kT} = \frac{N\mu_B^2 B}{kT}$

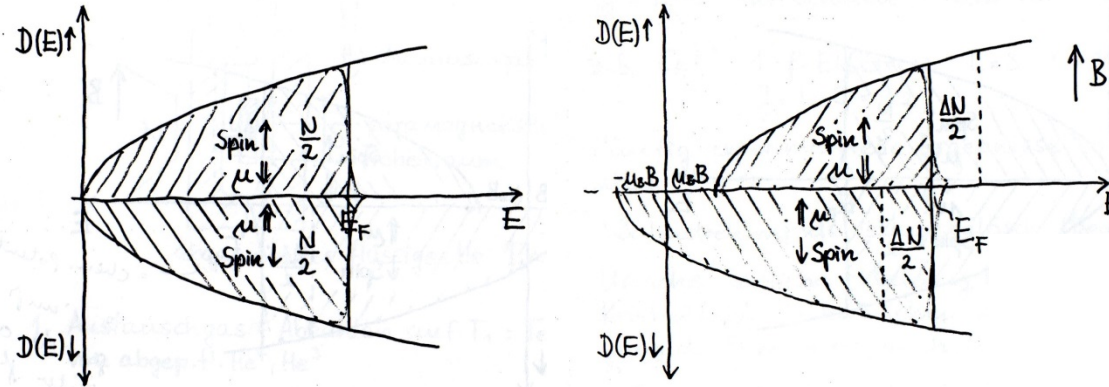
$$\chi_m = \frac{\mu_0 M}{B} = \frac{\mu_0 N\mu_B^2}{kT} \quad \text{Curie behaviour}$$

Experimentally one observes 1/100 of this value: → only electrons close to E_F can contribute to magnetism – ratio T/T_F

$$\chi_m = \frac{\mu_0 M}{B} = \frac{\mu_0 N\mu_B^2}{kT_F}$$

Density of states

$$B=0 : N(\uparrow) = N(\downarrow) = \frac{1}{2} N \quad D(E) \uparrow = D(E) \downarrow = \frac{1}{2} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \sqrt{E} \quad M=0$$



$$B>0 : N(\uparrow) - N(\downarrow) = \left(\frac{1}{2} N + \frac{1}{2} \Delta N \right) - \left(\frac{1}{2} N - \frac{1}{2} \Delta N \right) = \Delta N \quad M = \mu_B \Delta N \quad \Delta N = ???$$

$$D(E) \uparrow = \frac{1}{2} \int_{-\mu_B B}^{E_F} D(E + \mu_B B) dE = \frac{A}{3} (E_F + \mu_B B)^{3/2}$$

$$D(E) \downarrow = \frac{1}{2} \int_{-\mu_B B}^{E_F} D(E - \mu_B B) dE = \frac{A}{3} (E_F - \mu_B B)^{3/2}$$

$$A = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2}$$

$$\mu_B B \ll E_F \quad \mu_B B / kT \approx 0.001 \quad E_F / kT \approx 100$$

$$N_{\uparrow\downarrow} = \frac{A}{3} E_F^{3/2} (1 \pm \mu_B B / E_F)^{3/2} \approx \frac{A}{3} E_F^{3/2} (1 \pm \frac{3}{2} \mu_B B / E_F + \dots)$$

$$\Delta N = \frac{A}{3} E_F^{3/2} 3 \frac{\mu_B B}{E_F} = A \mu_B B E_F^{1/2}$$

$$M(T=0) = \frac{3}{2} N_e \left(\frac{\mu_B B}{E_F} \right) = N_e \mu_B \frac{3 \mu_B B}{2 k T_F}$$

$$\chi = \frac{3 \mu_B^2 B}{2 k T_F}$$

Pauli's spin susceptibility of conducting electrons

Additional effect due to B induced energetic splitting of electron levels below CB

→ Landau diamagnetism

$$\chi_{tot} = \chi_{para} + \chi_{dia} = \chi_{para} - \frac{1}{3} \chi_{para} = \frac{N_e \mu_B^2}{k T_F}$$

For $T > 0$

$$M(T) = M(0) \left[1 + \frac{\pi^2}{12} \left(\frac{T}{T_F} \right)^2 + \dots \right] \quad \rightarrow M \text{ is nearly independent from } T$$

Ferromagnetism

Appears below „Curie“ temperature T_c

At $T > T_c$ - paramagnetism

$$\chi = \frac{C}{T - T_c}$$

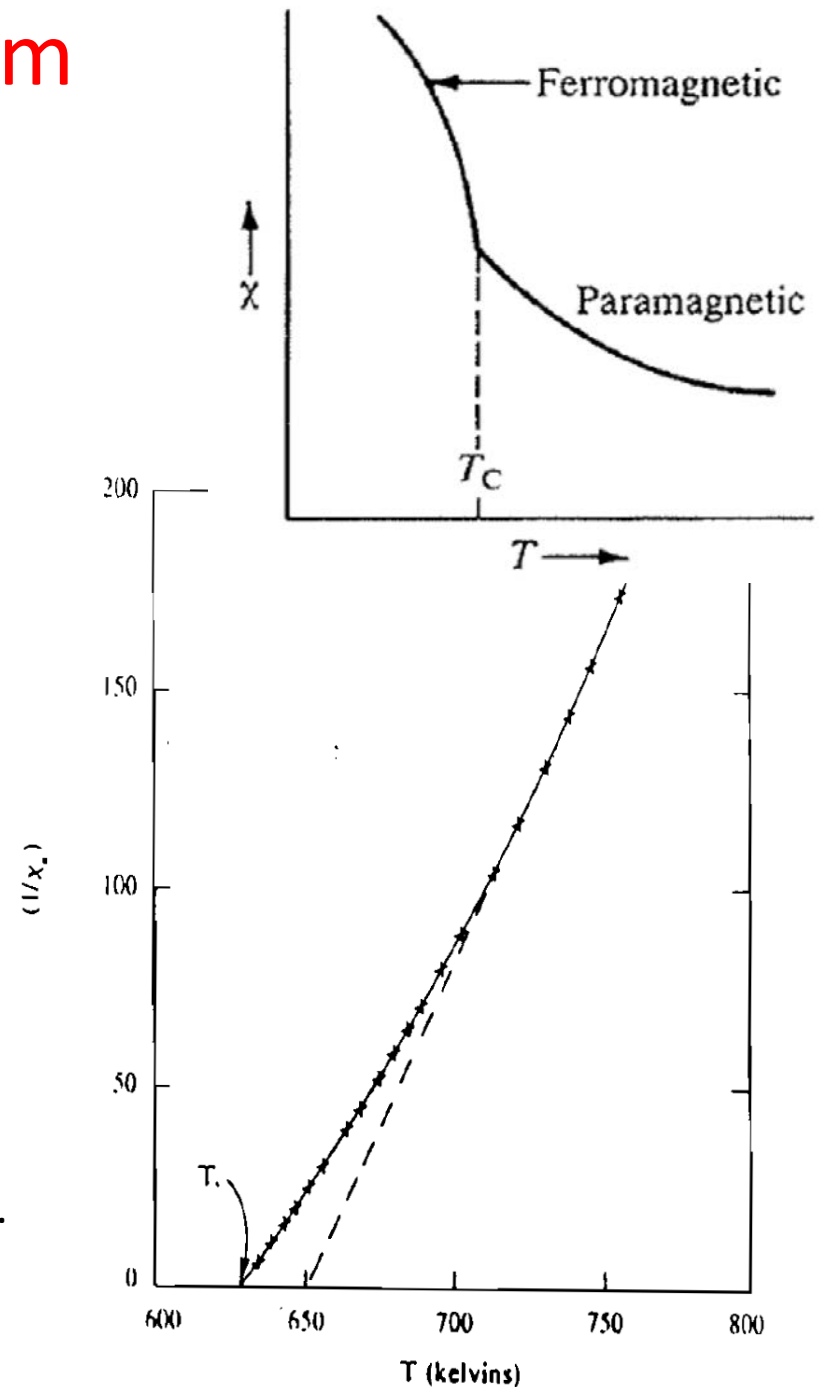
Curie- Weiss-
law

Typical ferromagnets :

Fe, Co, Ni – group 3d

Gd, Dy, - group 4f

Origin of ferromagnetism is the internal interaction spins caused by exchange fields between next neighbors or by “molecular field”.



Internal magnetic field

Weiß supposed the existence of internal magnetic field
The “molecular field”:

$$B_E = \lambda \mu_B M$$

Spin at one particular atom „is feeling“ magnetic moments of neighbored atoms, creating a „mean“ field B_E and small magnetisation M

$$\mu_0 M = \chi (B + B_E)$$

Using Curie law : $\chi_m = \frac{C}{T}$

$$\mu_0 M = \frac{C}{T} (B + \lambda \mu_0 M) = \frac{CB}{T - \lambda C}$$

Resulting in

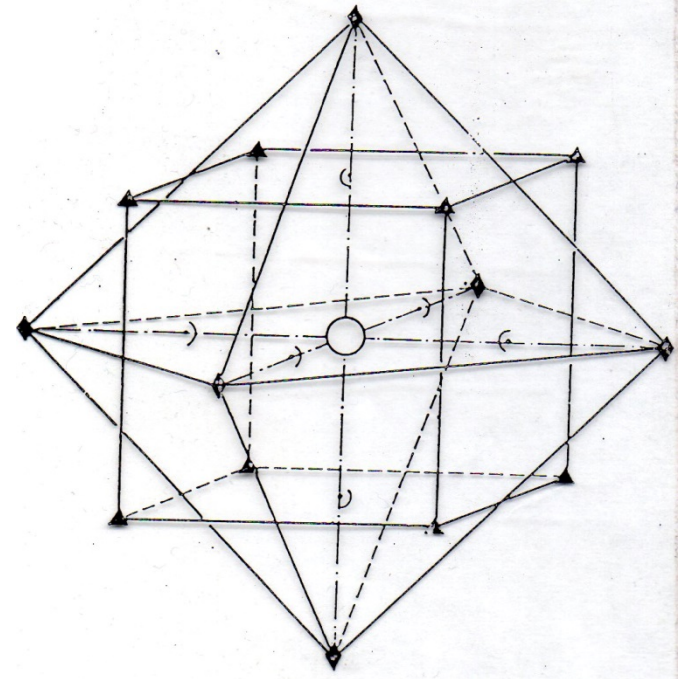
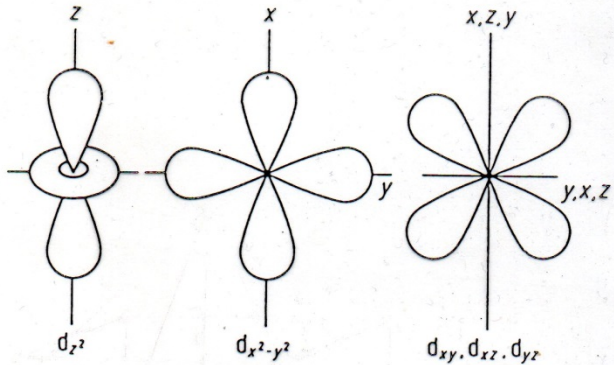
$$\chi_m = \frac{M}{B} = \frac{C}{T - \lambda C} = \frac{C}{T - T_c}$$

Providing singularity at $T=T_c$; Curie- Weiss law is well confirmed experimentally

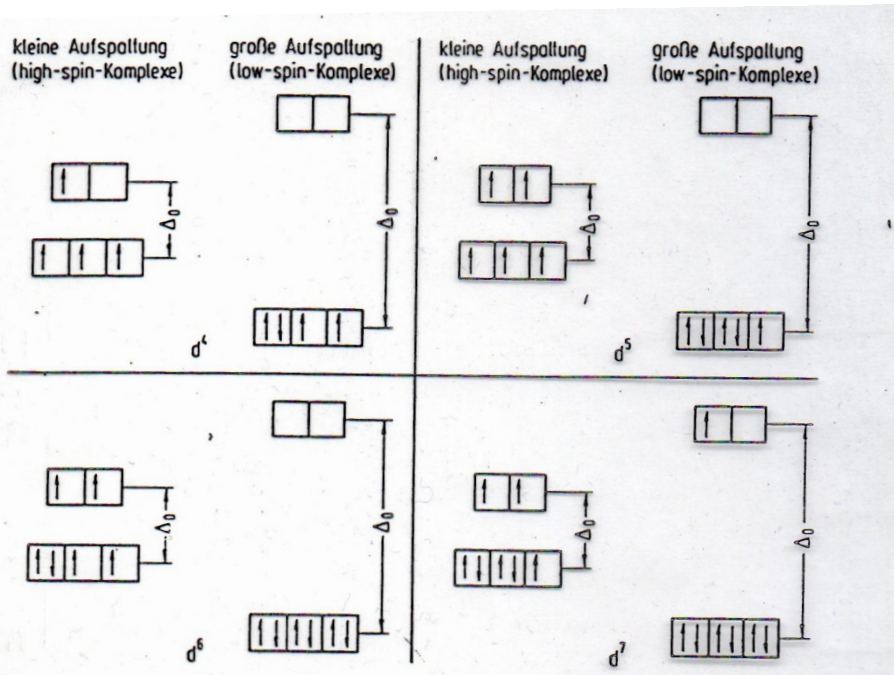
More advanced calculations predict : $\chi = \frac{C}{(T - T_c)^{4/3}}$ $C = \frac{NJ(J+1)g^2 \mu_B^2}{3k}$

For Fe $T_c = 1000\text{K}$, $\lambda = 5000$; saturation $M_s = 1700 \text{ Gau\ss} \rightarrow \lambda * M_s = 10^7 \text{ Gau\ss} = 10^3 \text{ T} !!!$
 $T < T_c$, M_s becomes T-dependent 1T = 10000 Gau\ss

Crystal field



For example : FeN_6 complex



Explanation by Brillouin function

$T < T_c$, M_s becomes T -dependent, behavior explained by evaluation of Brillouin function,

external field $B=0$, only molecular field

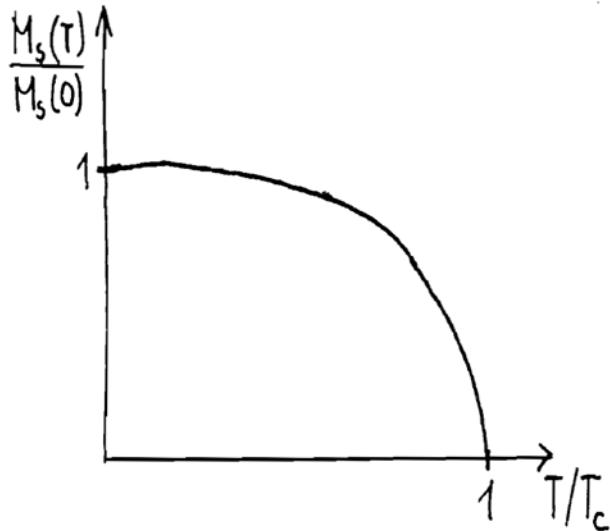
$$B_E = \lambda \mu_B M \quad M = N\mu \tanh \frac{\mu \mu_0 \lambda M}{kT}$$

Solution for M $0 < T < T_c$: determine crossing point of $M = \frac{kT}{\mu \mu_0 \lambda} x$ with $M = N\mu \tanh(\mu B / kT)$

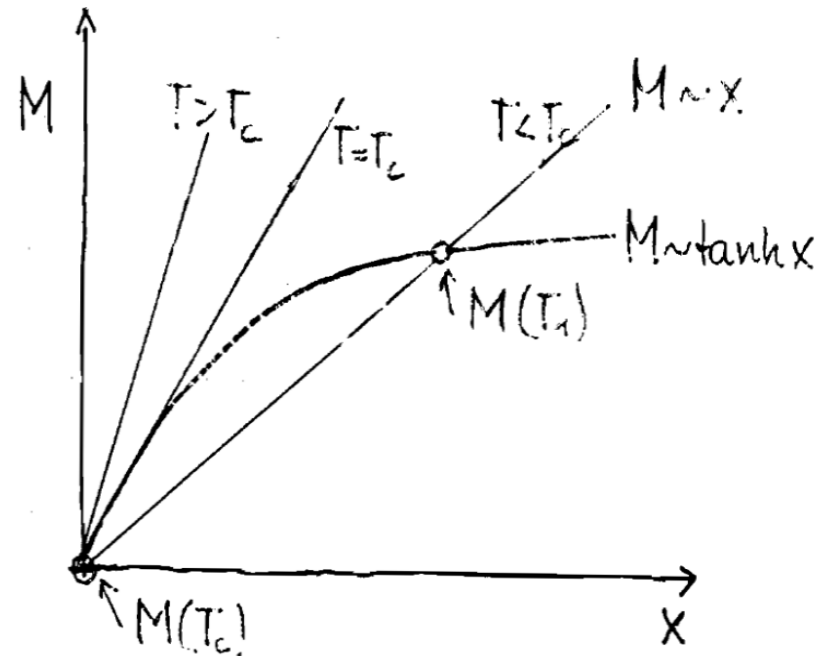
with $x = \frac{\mu \mu_0 \lambda M}{kT}$

Slope $M(x)$ increases for increasing T

→ $M(T)/M(0)=1$ at $T=0$ and zero at $T=T_c$.



$T_c = 1000\text{K}$, $N = 10^{29} / \text{m}^3 \rightarrow \lambda = 10^4$



Origin of internal field is "molecular field: an atom with unpaired spin is surrounded by other atoms in certain geometrical arrangement"

Exchange interaction

Depending on next neighbor distances the energy separation between the bonding and antibonding orbitals changes and results in paired or mainly unpaired arrangement, Phase transition between different magnetization can be induced by local distortions of the crystal field.

Exchange interaction is expressed by Heisenberg exchange energy $U = -2J\hat{S}_i\hat{S}_j$

where J is exchange integral, ferromagnetic coupling: $J>0$, antiferromagnetic coupling: $J<0$,

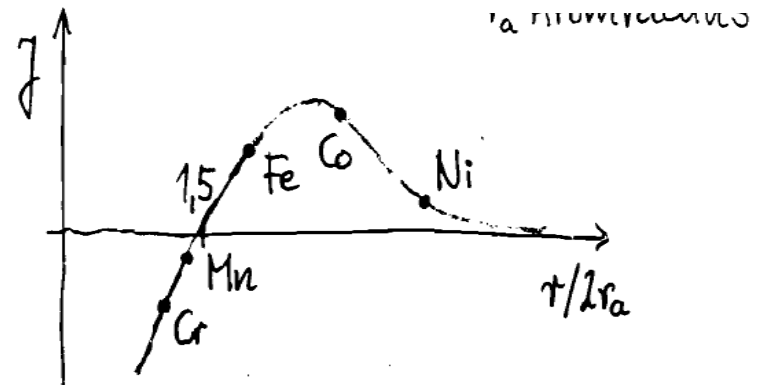
$$J = \frac{\mu^2\mu_0 N}{Z} \lambda \quad \rightarrow \text{See lecture for QM approach}$$

J cannot be explained by dipole-dipole interaction but interplay between kinetic energy and Coulomb energy \rightarrow Exchange interaction,

For metals like Fe, Co, Ni...

Slater criterion expresses J as function of $r/2r_a$, where r is next neighbor distance and r_a the atomic radius.

$\rightarrow J>0$ for $r/2r_a > 1,5$ and $J<0$ for $r/2r_a < 1,5$.



Nature of spin exchange interaction

interaction between two neighbored spins

$$\hat{H} = \bar{A} \hat{S}^a \hat{S}^b$$

eigenvalues we compose the total spin :

$$\vec{S} = \hat{S}^a + \hat{S}^b$$

Calculated by relation

$$\hat{S}^2 = (\vec{S}^a)^2 + (\vec{S}^b)^2 + 2\hat{S}^a \hat{S}^b$$

Possible eigenvalues for \hat{S}^2 using $S(S+1)$ are 0 ($S=0$) and 2 ($S=1$).

Thus eigenvalues of $\hat{S}^a \hat{S}^b$ are $\frac{3}{4}$.

Therefore the eigenvalues of $\hat{S}^a \hat{S}^b$ are $\frac{1}{4}$ (for $S=1$) and $-\frac{3}{4}$ (for $S=0$).

Subsequently the eigenvalues of Hamiltonian are $E = \frac{1}{4} A$ ($S=1$) and $E = -\frac{3}{4} A$ ($S=0$).

Each energy level is $2S+1$ times degenerated, therefore $S=0$ is a singulett and $S=1$ is a triplett state. m_s component of total spin is 0 for singulett and ($-1, 0, +1$) for triplett.

The spin eigenstates are

	S	ms	Spin eigenvalue $\chi_{S,T}$	$S^a S^b$
Triplett	1	1	$ \uparrow\uparrow\rangle$	$\frac{1}{4}$
	1	0	$(\uparrow\downarrow\rangle + \downarrow\uparrow\rangle)/\sqrt{2}$	$\frac{1}{4}$
	1	-1	$ \downarrow\downarrow\rangle$	$\frac{1}{4}$
Singulett	0	0	$(\uparrow\downarrow\rangle - \downarrow\uparrow\rangle)/\sqrt{2}$	$-\frac{3}{4}$

Eigenvalues of $\hat{S}^a \hat{S}^b$ by linear combination of electron basic wave functions $\psi_a(r_1)$ and $\psi_b(r_2)$

To the total spatial wave function $\psi_a(r_1)\psi_b(r_2)$ for singulett and triplett states

$$\psi_S = \frac{1}{\sqrt{2}}[\psi_a(r_1)\psi_b(r_2) + \psi_a(r_2)\psi_b(r_1)]\chi_S$$

$$\psi_T = \frac{1}{\sqrt{2}}[\psi_a(r_1)\psi_b(r_2) - \psi_a(r_2)\psi_b(r_1)]\chi_T$$

$\chi_{S,T}$ Spin function

Energy of singulett and triplett states are

$$E_S = \int \psi_S^* \hat{H} \psi_S dr_1 dr_2 \quad \text{-----} \quad E_T = \int \psi_T^* \hat{H} \psi_T dr_1 dr_2$$

The difference is : $E_S - E_T = 2 \int \psi_a^*(r_1)\psi_b^*(r_2)\hat{H}\psi_a(r_2)\psi_b(r_1)dr_1 dr_2 = 2J$

Which is the exchange integral J. The total Hamiltonian is

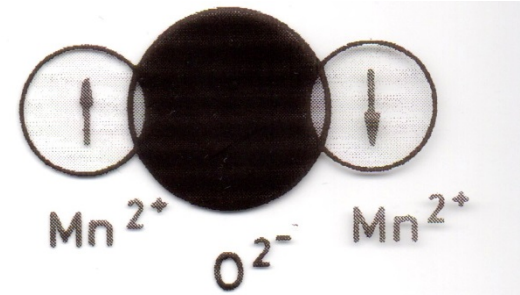
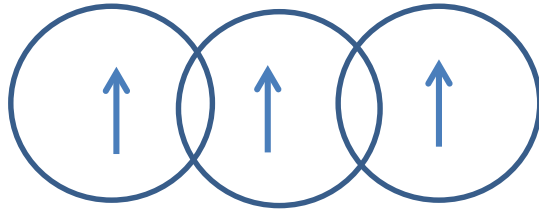
$$\hat{H} = \frac{1}{4}(E_S + 3E_T) - (E_S - E_T)\hat{S}^a \hat{S}^b = \frac{1}{4}(E_S + 3E_T) - \hat{H}^{spin}$$

Where the spin dependent part is : $\hat{H}^{spin} = -2J\hat{S}^a \hat{S}^b$

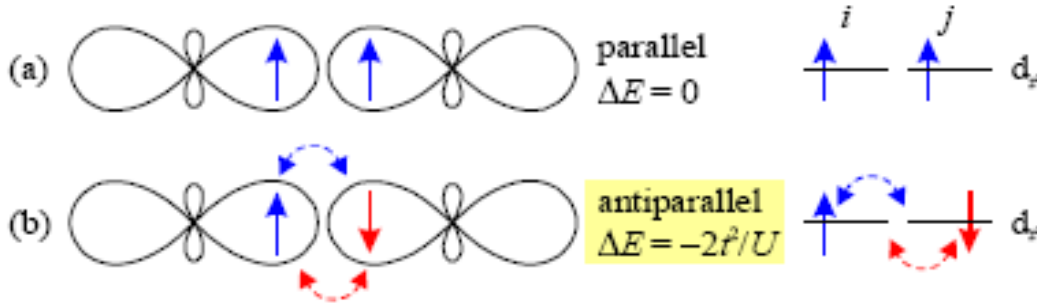
For triplett state $E_S > E_T$ $J > 0$ For singulett state $E_S < E_T$ $J < 0$

Kinds of magnetic exchange interaction

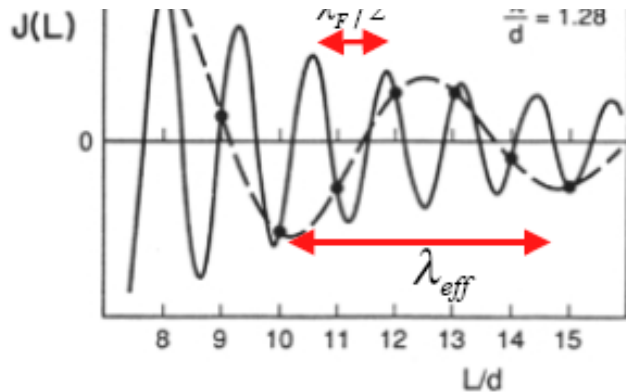
Direct exchange interaction



Indirect exchange interaction



superexchange



$$\lambda_{\text{eff}} = \frac{2\pi}{|2k_F - 2\pi/d|}$$

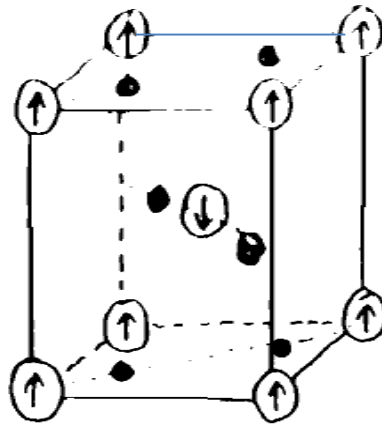
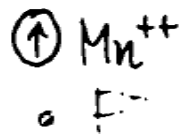
$$= \frac{d\lambda_F}{|d - \lambda_F|}$$

Oscillatory interaction via electrons in CB, proposed by Rudermann, Kittel, Kasuya, Yoshida = RKKY interaction

Antiferromagnetism $J < 0$



Z.B. MnF_2

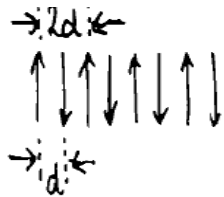


$$\chi = \frac{C}{(T + \Theta)}$$

AF magnetism vanishes above T_N – Neel temperature

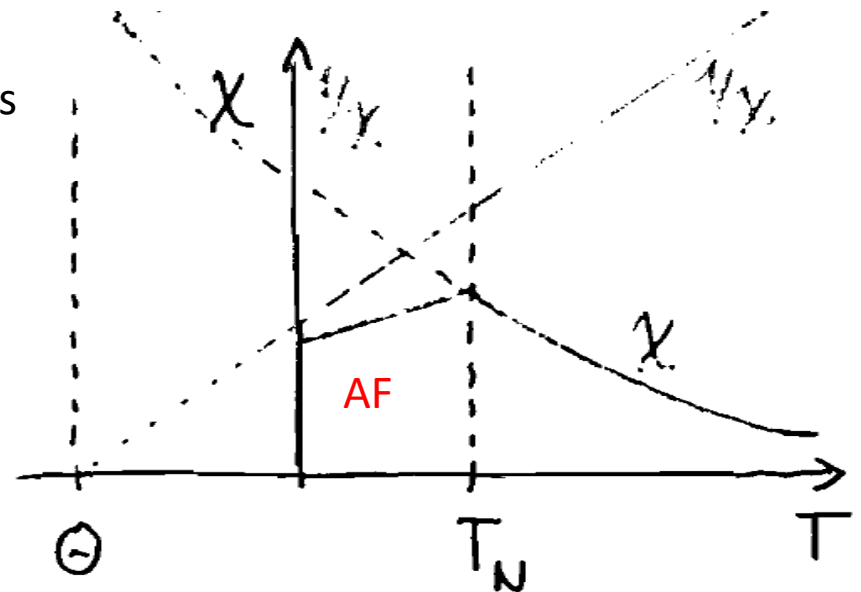
$$T_N = \kappa M$$

$\chi_m \approx \frac{1}{T}$ curve shifted to “negative” Curie-Weiß temperature Θ . In range $0 > T > T_N$ the total magnetization is zero and AF order increases



Experimental evidence by spin resolved neutron scattering

is the antiferromagnetic x-change interaction



Antiferromagnetism

Spins are ordered antiparallel arrangement at temperature below certain ordering temperature - Neel temperature T_N

Suceptibility at $T=T_N$ is not infinite but shows certain cusp.

supposing

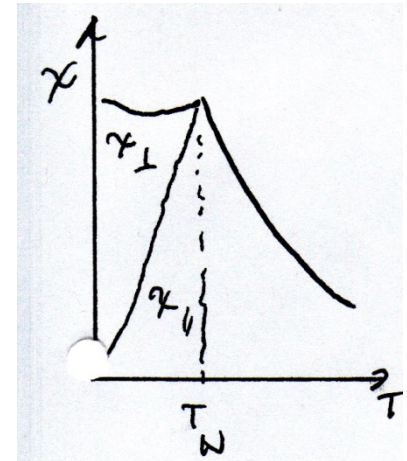
$$T_N = \kappa M$$

$T > T_N$

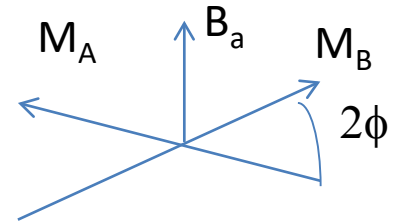
$$\chi = \frac{2CT - 2\kappa C^2}{T^2 - (\kappa C)^2} = \frac{2C}{T + T_N} \quad \text{C - Curie constant}$$

In experiment

$$\chi = \frac{C}{(T + \Theta)}$$



Below T_N sucseptibility differs for $B \perp \uparrow\downarrow$ and $B \parallel \downarrow\uparrow$



For $B_a \perp \uparrow\downarrow$ Energy density is $u = \mu M_A M_B - B_a (M_A + M_B) \approx -\mu M^2 [1 - \frac{1}{2}(2\phi)^2] - B_a 2M\phi$

$$\frac{\partial u}{\partial \phi} = 0 = 4\mu M^2 \phi - 2B_a M$$

$$\phi = \frac{B_a}{2\mu M} \quad \chi_{\perp} = \frac{2M\phi}{B_a}$$

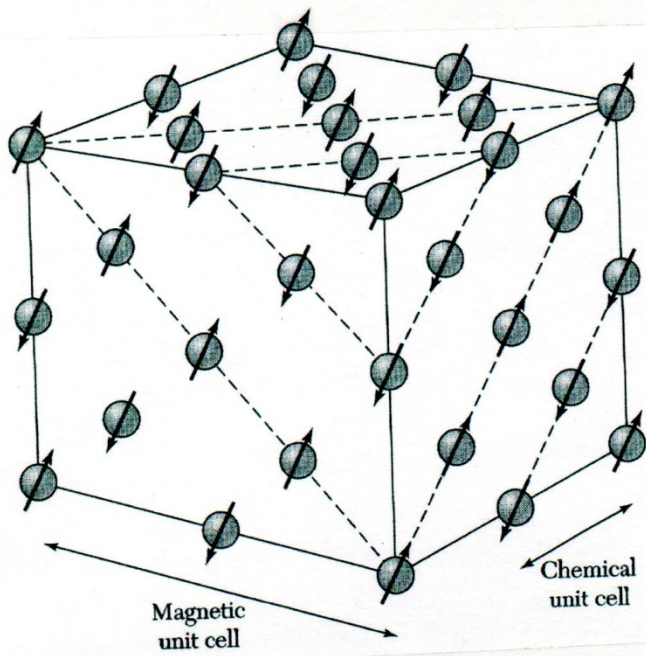
For $B_a \parallel \uparrow\downarrow$

$$\chi_{\parallel} = 0$$

at $T = \Theta$, due spin fluctuations χ_{\parallel} increases with $T < T_N$

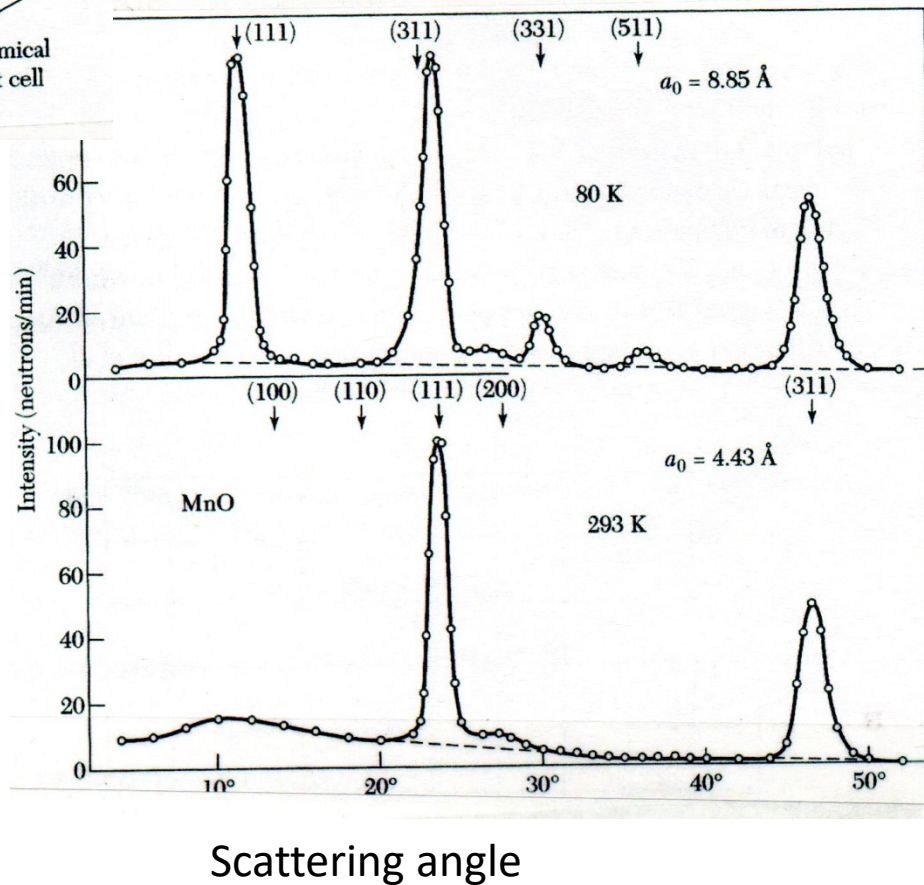
Table 2 Antiferromagnetic crystals

Substance	Paramagnetic ion lattice	Transition temperature, T_N , in K	Curie-Weiss θ , in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF ₂	bc tetr.	67	82	1.24	0.76
FeF ₂	bc tetr.	79	117	1.48	0.72
FeCl ₂	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
CoCl ₂	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
NiCl ₂	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			



Spin arrangement in Mn^{2+} in MnO ,
 Oxygen atoms are not shown \rightarrow
 indirect spin exchange

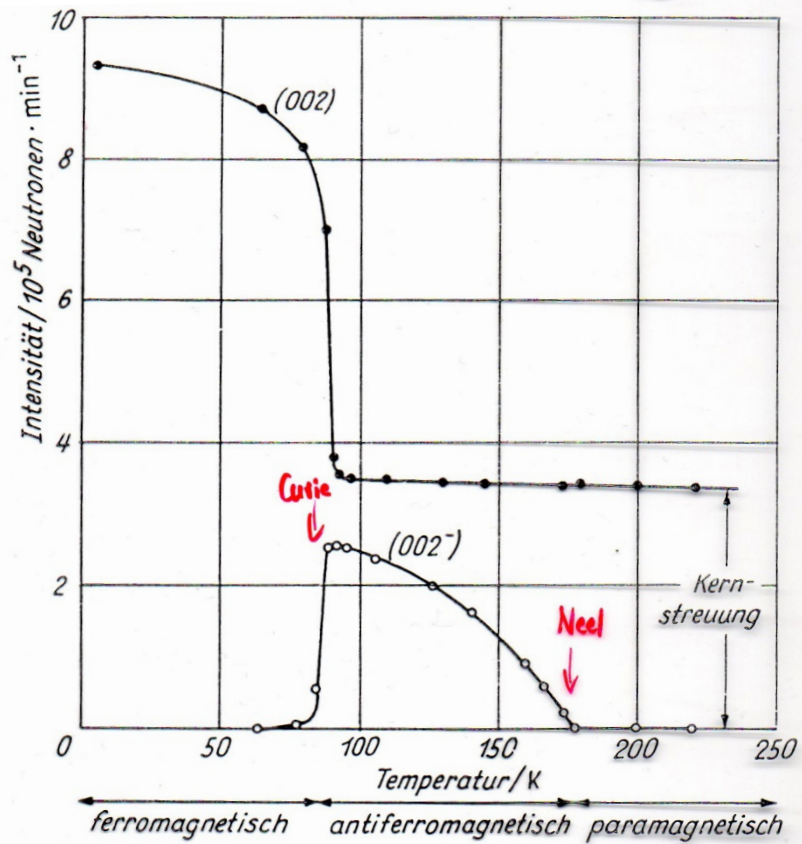
T-dependent neutron
 diffraction at MnO



*Antiferro-
 magnetic
 phase*

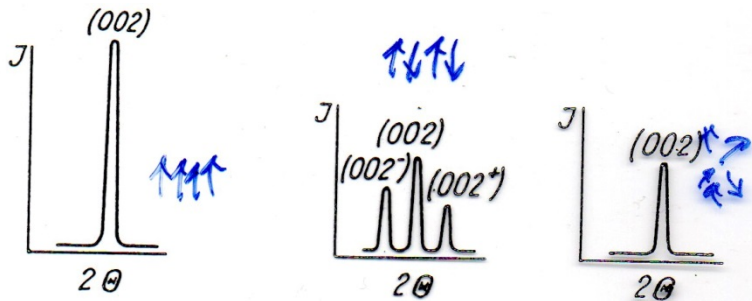
*paramagnetic
 phase*

Scattering angle



Neutron diffraction at bcc FeRh showing transition from Ferromagnetic phase – via antiferromagnetic phase to paramagnetic phase

$T_N = 350 \text{ K}$, $T_C = 675 \text{ K}$



Spin waves - Magnons

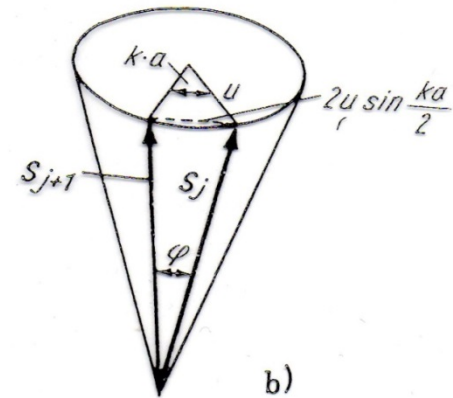
Ferromagnetic ground state $\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

Possible excitation $\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow$ Costs $\Delta E = 8JS^2$

Compromise : spin flip distributed over many neighbored spins

→ Spin precession

Spin wave



$$E_{exc} = -\frac{N}{2} 2J(S_J S_{J+1} + S_J S_{J-1} + \dots) = -2NAS^2$$

with $\Delta E = 8AS^2$ per spin

Dispersion relation for magnons :

$$\hbar\omega = 4JS[1 - \cos(ka)]$$

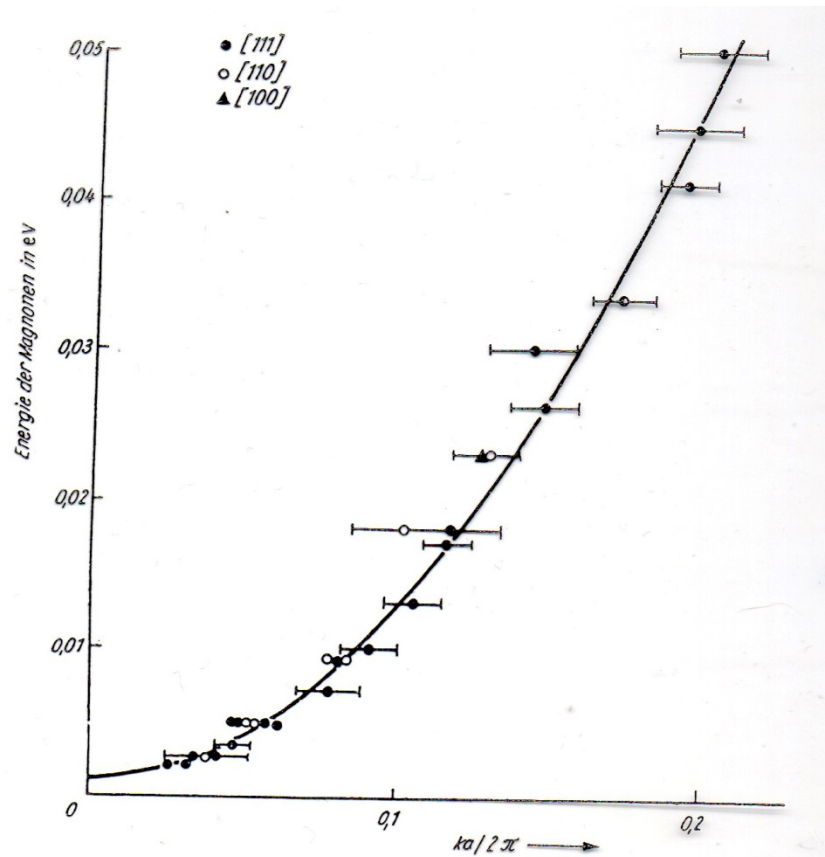
$ka \ll 1 \rightarrow 1 - \cos(ka) \rightarrow \frac{1}{2} k^2 a^2$

$$\hbar\omega = 4JSa^2 k^2 \approx Dk^2$$

Compare phonons : $\omega \approx k$

D determined by neutron scattering

Magnon spectrum of fcc Co alloy



Thermal excitation of magnons: (for bosons) number of magnons excited n_k

$$n_k = \frac{1}{\exp(\hbar\omega_k / kT) - 1}$$

Total number of magnons excited at temperature T

$$\sum_k n_k = \int d\omega D(\omega) \langle n(\omega) \rangle$$

$D(\omega)$ is magnon density of modes, integrated over 1st Brillion zone

$$\frac{d\omega}{dk} = \frac{4JSa^2k}{\hbar} = 2\left(\frac{2JSa^2}{\hbar}\right)^{1/2} \omega^{1/2}$$

$$D(\omega) = \frac{1}{4\pi^2} \left(\frac{\hbar}{2JSa^2}\right)^{3/2} \omega^{1/2}$$

magnon density of modes is

So that total number of magnons is

$$\sum_k n_k = \frac{1}{4\pi^2} \left(\frac{\hbar}{2JSa^2}\right)^{3/2} \int_0^\infty d\omega \frac{\omega^{1/2}}{\exp(\hbar\omega/kT) - 1} = \frac{1}{4\pi^2} \left(\frac{kT}{2JSa^2}\right)^{3/2} \int_0^\infty dx \frac{x^{1/2}}{e^x - 1}$$

$= 0.0587 * 4\pi^2$

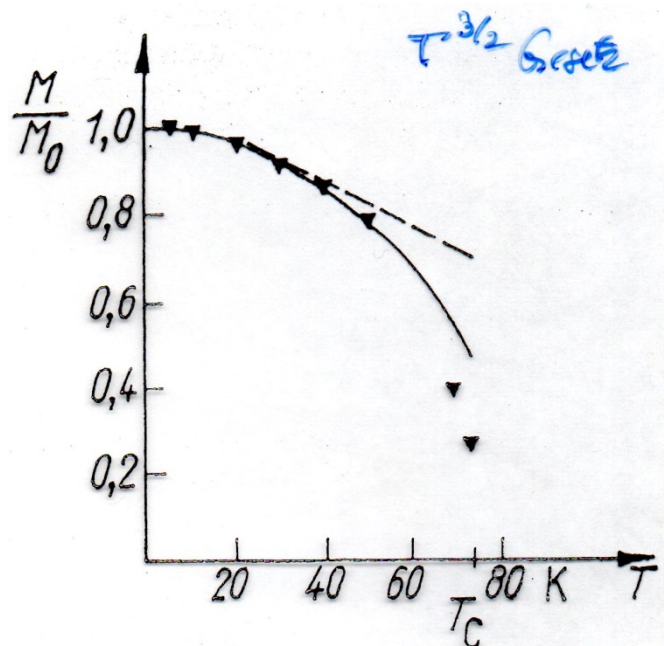
Number N/atom/unit volume is Q/a^3 with $Q=1,2,4$ for sc, bcc, fcc

$$\frac{\sum_k n_k}{NS} = \frac{\Delta M}{M(0)} = \frac{0.05872C}{SQ} \left(\frac{kT}{2JS}\right)^{3/2} \quad \text{Bloch } T^{3/2} \text{ law}$$

Number N/atom/unit volume is Q/a^3 with $Q=1,2,4$ for sc, bcc, fcc

$$\frac{\sum_k n_k}{NS} = \frac{\Delta M}{M(0)} = \frac{0.05872C}{SQ} \left(\frac{kT}{2JS}\right)^{3/2}$$

Bloch $T^{3/2}$ law



Ferromagnetic magnons $J>0$; antiferromagnetic Magnons : $J<0$

Ferrimagnetic magnons have 2 magnetic moments/ uc \rightarrow two branches

Magnetic domains and hystereses

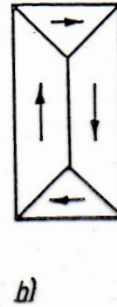
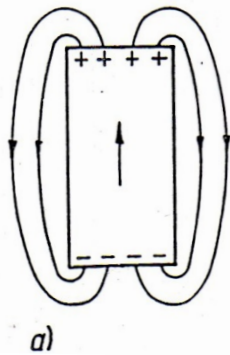
At $T \ll T_c$ all magnetic moments aligned, but amcroscopic moment can be zero

Origin: solid shows magnetic domains = Weiss domains

Separation into domains is energetically favoured

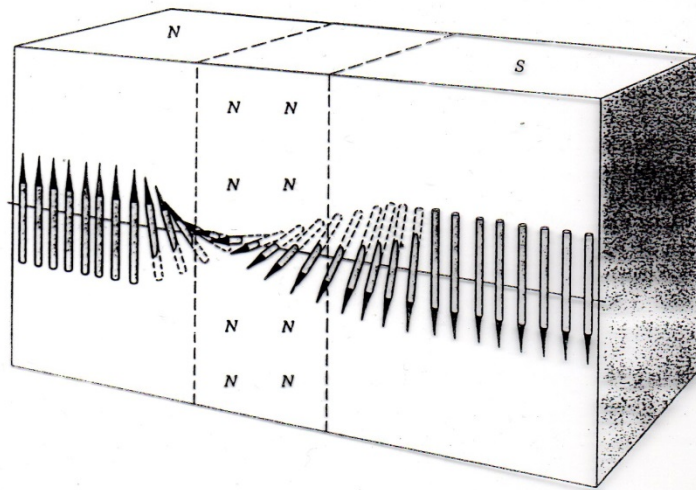
Large magn energy

Magn. Lines outside solid



Low magnetic energy

Magnetic lines are closed inside solid

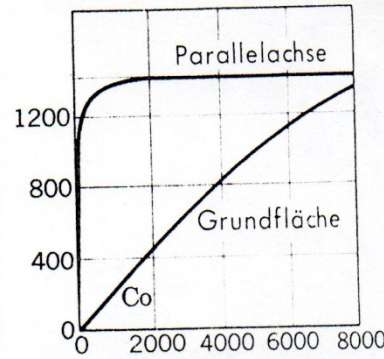
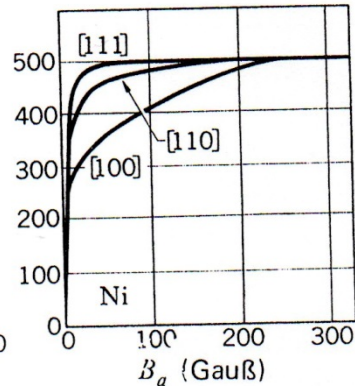
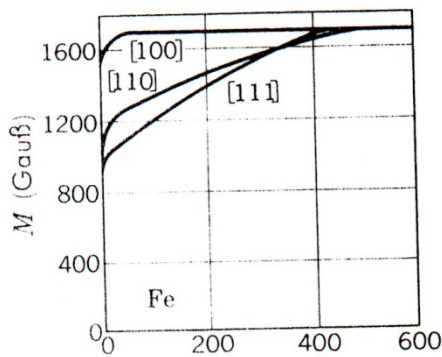


Magnetic domains are separated by **Bloch walls**

Thickness amount many unit cells
In Fe : about $300a$

Spin orientation rotates by application of external B-field

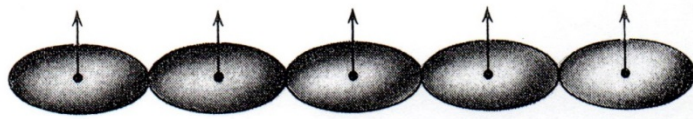
Exchange energy differs for crystal direction



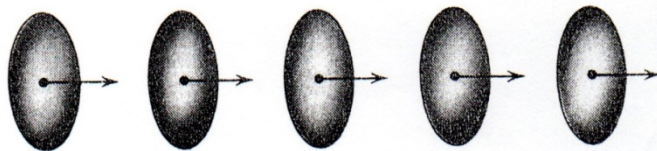
„hard“ and „weak“ directions

Fe: [100] „weak“
[111] „hard“

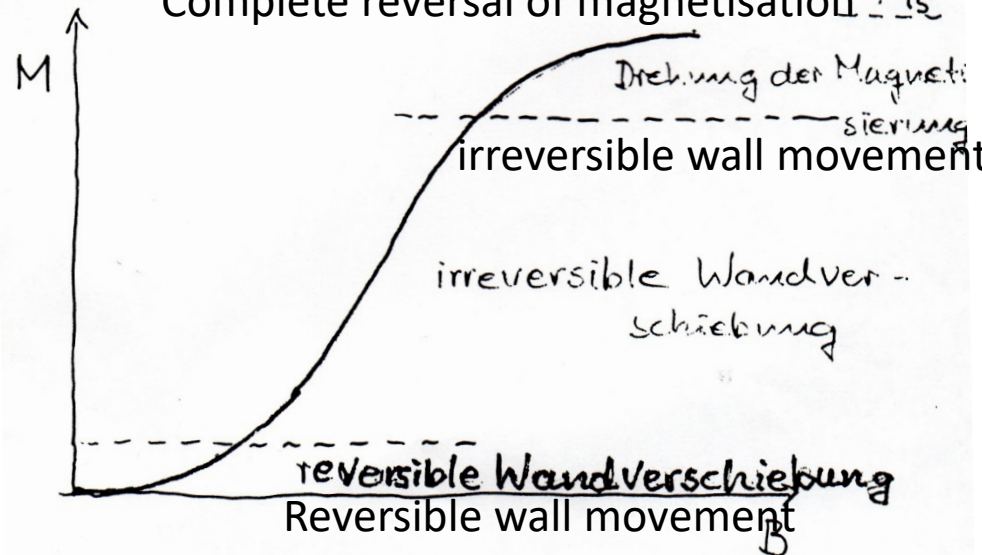
Ni [100] „hard“
[111] „weak“



(a)



Complete reversal of magnetisation M_s



Ferromagnet in external magnetic field

In weak field domains in optimum spindirection to external field grow in size compared to domains of other spin orientation

→ domain wall displacement typically over weak direction

In strong field: all domains rotate into field direction – also via „hard“ direction

Hysteresis loop

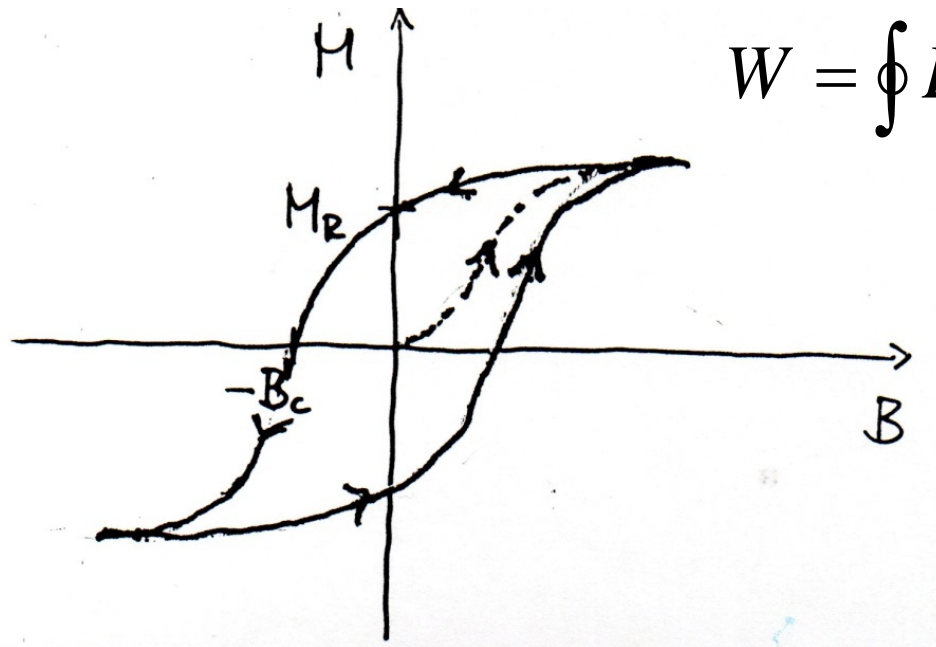
Power loss by one cycle

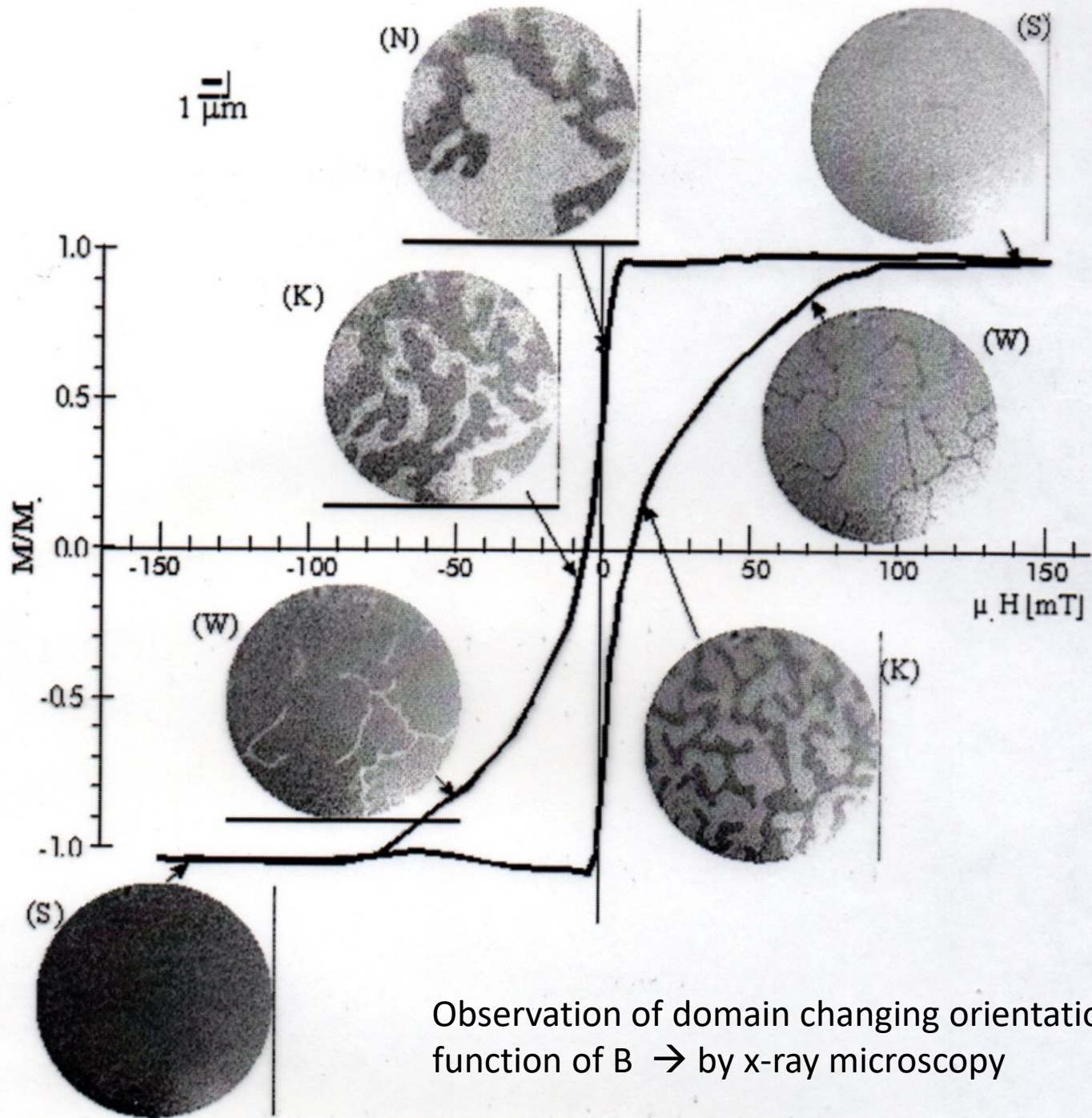
B_c – coercitive force

Field apply to cancel macroscopic magnetization

M_R – remanescence

Macroscopic magnetization at $B=0$





Observation of domain changing orientations as function of $B \rightarrow$ by x-ray microscopy

Magnetic materials

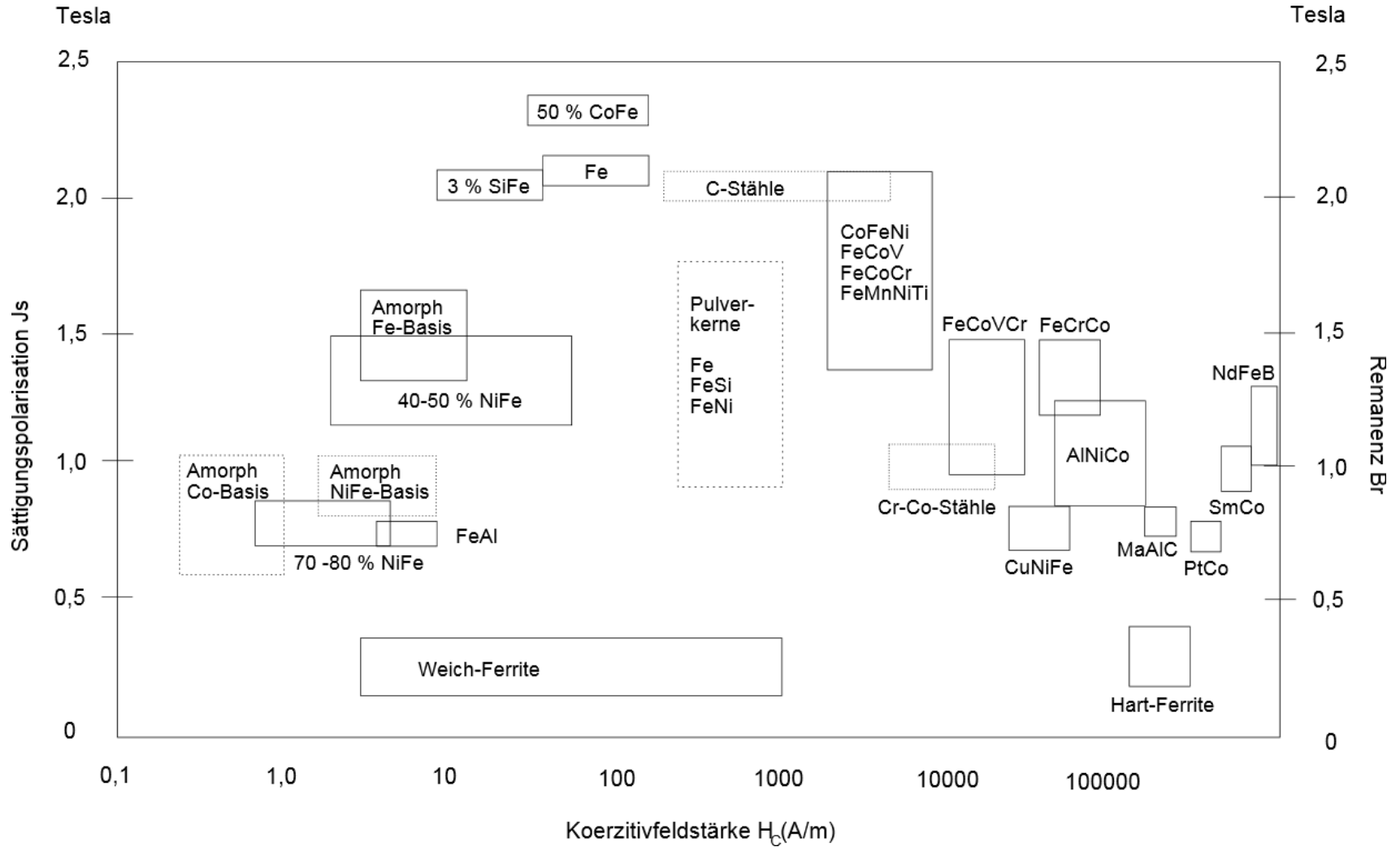


Tabelle N 3

Einige Eigenschaften weichmagnetischer Materialien *)

Curie Temp.
↓

Material	Zusammensetzung Gew. %	Wärmebehandlung **) °C	μ_A/μ_0	μ_{\max}/μ_0	H_c/Am^{-1} ($B = 0$)	$I_S/\text{Wb m}^{-2}$	$T_C/^\circ\text{C}$
Eisen	99,8	950	150	5000	80	2,15	770
Eisen (gerein.)	99,95	1480 (H_2); 880	10000	200000	4	2,15	770
Silizium-Eisen	4 Si	800	500***)	7000	40	1,97	690
Alperm	16 Al	600 (A)	3000	55000	3,2	0,80	400
78 Permalloy	78,5 Ni	1050; 600 (A)	8000	100000	4	1,08	600
Supermalloy	5 Mo, 79 Ni	1300 (H_2 , K)	100000	1000000	0,16	0,79	400
Mu-Metall	5 Cu, 2 Cr, 77 Ni	1175 (H_2)	20000	100000	4	0,65	
Hipernik	50 Ni	1200 (H_2)	4000	70000	4	1,60	500
Permendur	50 Co	800	800	5000	160	2,45	980
45-25 Perminvar	25 Co, 45 Ni	1000; 400	400	2000	95	1,55	715
7-70 Perminvar	7 Co, 70 Ni	1000; 425	850	4000	48	1,25	650
Co	99 Co	1000	70	250	796	1,79	1120
Ni	99 Ni	1000	110	600	56	0,61	358

*) Nach BOZORTH; bezüglich hartmagnetischer Materialien vgl. Tab. N 4

) A: abgeschreckt; H_2 : getempert in reinem Wasserstoff; K: kontrollierte Abkühlungsgeschwindigkeit*) $B = 2 \cdot 10^{-3} \text{ Wb/m}^2$ 