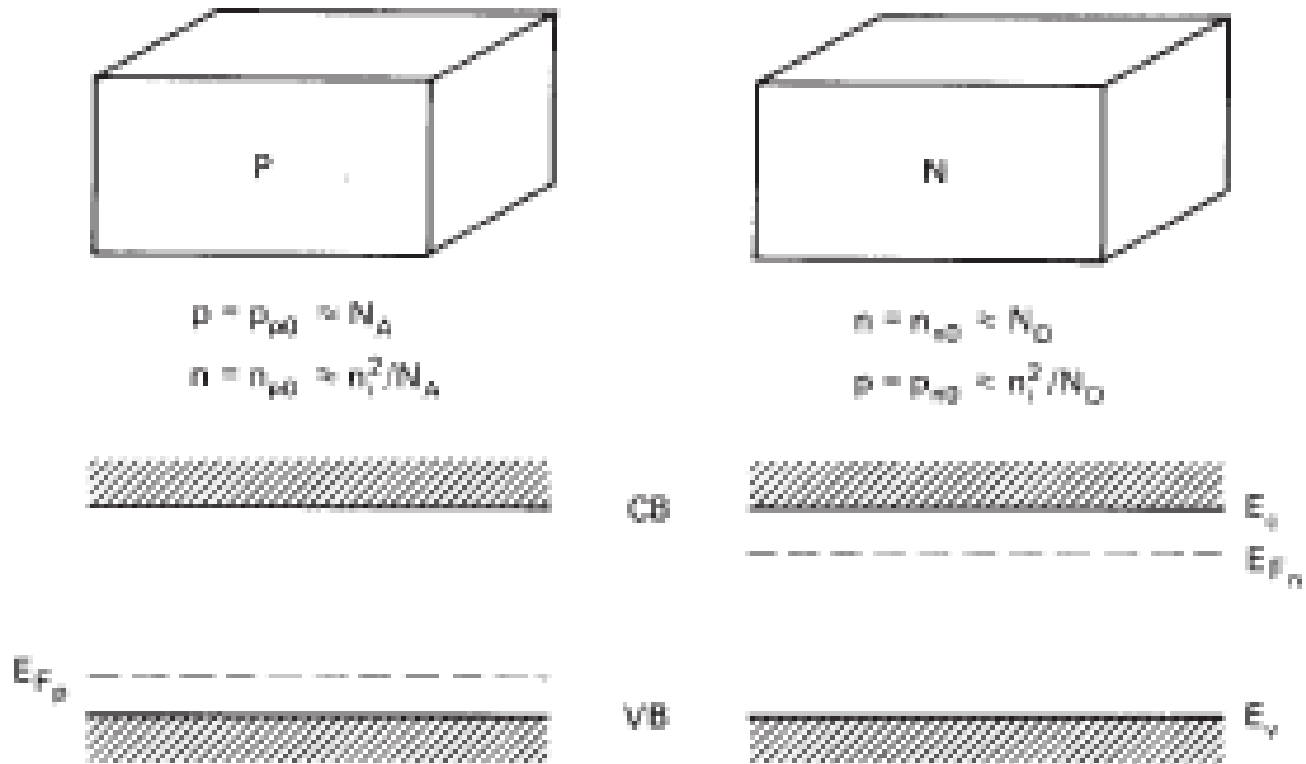
Solid state physics for Nano

Lecture 7: pn junctons

Prof. Dr. U. Pietsch

pn-junction



Elements and principal band structure of pn- junction elements

Basis parameters

$$n_n p_n = n_p p_p = n_i^2$$

p_p – hole concentration at p-site - majority
p_n - hole concentration on n-site - minority

Due to charge carrier diffusion current, j_{Diff}, ionized (fixed) ions are left (A^- and D^+) originating an electric field, and subsequently drift current, j_{Drift} opposite to j_{Diff} of mobile charge carriers:
in equilibrium $j_{Diff} = - j_{Drift}$ → $j_{total} = 0$

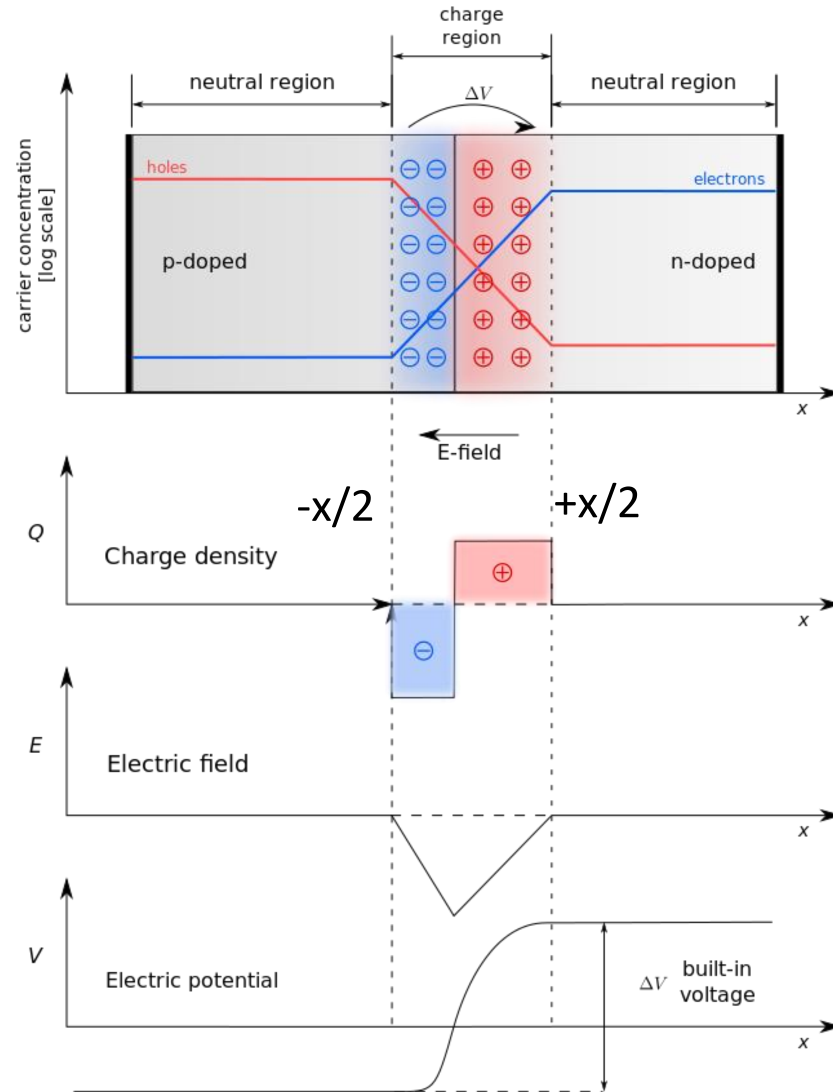
Originates contact potential: for hole current

$$\frac{dV_0}{dx}\frac{e}{kT} = -\frac{1}{p}\frac{dp}{dx}$$

V_p – potential on p site
V_n – potential on n-site

$$V_0 = \frac{kT}{e}\ln\frac{p_p}{p_n}$$

$p_p >> p_n$ → V>0



Because of $p_p=N_A$ and $p_n = n_i^2/N_D$,

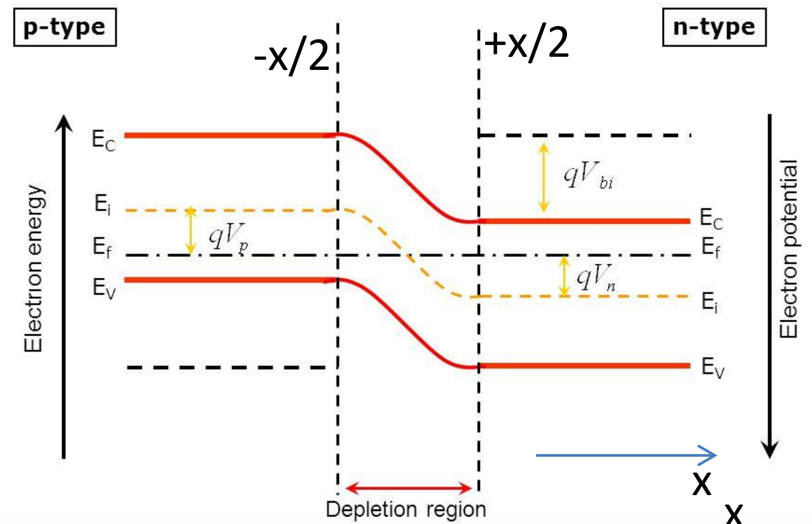
$$V_0 = \frac{kT}{e}\ln\frac{p_p}{p_n} = \frac{kT}{e}\ln\frac{N_D N_A}{n_i^2} = \frac{kT}{e}\ln\frac{n_n}{n_p}$$

$$\frac{d^2V}{dx^2} = \begin{cases} -\dfrac{eN_A}{\varepsilon\varepsilon_r} \quad 0 < x_p < \dfrac{x}{2} \\ +\dfrac{eN_D}{\varepsilon\varepsilon_r} \quad -\dfrac{x}{2} < x_n < 0 \end{cases}$$

Boundary conditions: V=0; dV/dx=0 at $x_{n,p}=|x/2|$

$$V_0(x) = -\frac{e}{2\varepsilon\varepsilon_r}(N_D x_n^2 + N_A x_p^2)$$

$$x_n = \sqrt{\frac{2V_0\varepsilon\varepsilon_r}{e}\frac{N_A}{N_D}\frac{1}{N_A+N_D}}; \quad x_p = \frac{N_D}{N_A}x_n$$



$$E(x) = -gradV(x) = \frac{e}{\varepsilon\varepsilon_r}(N_D x_n + N_A x_p)$$

$$E_n(x) = -\frac{eN_D}{\varepsilon\varepsilon_r}x_n;....$$

$$x = \propto \sqrt{\frac{1}{N}}$$

Diffusion and drift current

$J = j_n + j_p$; electrons diffuse n→ p; and holes from p→n; stationary case d/dt=0

$$\frac{dj_p}{dx} = -\frac{dj_n}{dx} = -eU$$

$$j_n = eD_n \frac{dn}{dx} - e\mu_n n \frac{dV}{dx}$$

$$j_p = -eD_p \frac{dp}{dx} + e\mu_p p \frac{dV}{dx}$$

Diffusion Drift

For minority charge carriers neglect drift current

$$j_n^{(p)} = eD_n \frac{dn}{dx}; __ j_p^{(n)} = -eD_p \frac{dp}{dx}$$

For majority charge carriers neglect diffusion current

$$j_n^{(n)} = -e\mu_n n \frac{dV}{dx}; __ j_p^{(p)} = e\mu_p p \frac{dV}{dx}$$

Outside space charge diffusion current depends on drift current of minority charge carriers because here → dV/dx=0

$$\frac{d^2p}{dx^2} = -\frac{1}{eD_p}\frac{dj_p^{(n)}}{dx} = \frac{V}{D_p} = \frac{p-p_n}{L_p^2}; \qquad \frac{d^2n}{dx^2} = \frac{n-n_p}{L_n^2}$$

Generell solution

$$p = p_n + A_1 \exp(x/L_p) + A_2 \exp(-x/L_p)$$

$$n = n_p + B_1 \exp(x/L_n) + B_2 \exp(-x/L_n)$$

$p \to p_n$ for $x \to +\infty$ and $n \to n_p$ for $x \to -\infty$; therefore $A_1 = B_2 = 0$

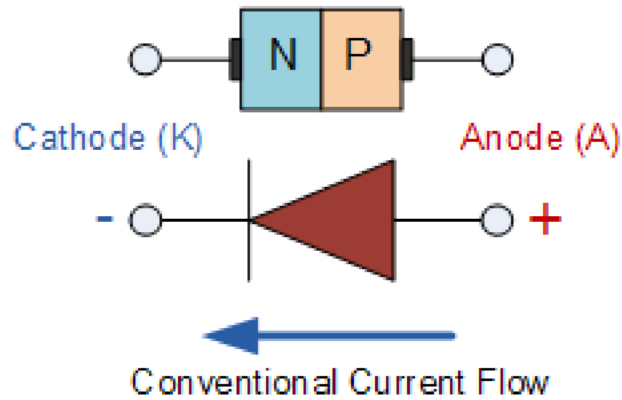
From Poisson equation: $A_2 = p_n(e^{\frac{eU}{kT}} - 1)e^{x/L_p}; \qquad B_1 = n_p(e^{\frac{eU}{kT}} - 1)e^{-x/L_n}$

$$p(x) = p_n + p_n[e^{eU/kT} - 1]e^{(x_n - x)/L_p}; \qquad n(x) = n_p + n_p[e^{eU/kT} - 1]e^{(x - x_p)/L_n}$$

$$j_p^{(n)}(x) = e\frac{D_p}{L_p}p_n[e^{eU/kT} - 1]e^{(x_n - x)/L_p}; \qquad j_n^{(p)}(x) = \frac{eD_n}{L_n}n_p[e^{eU/kT} - 1]e^{(x - x_p)/L_n}$$

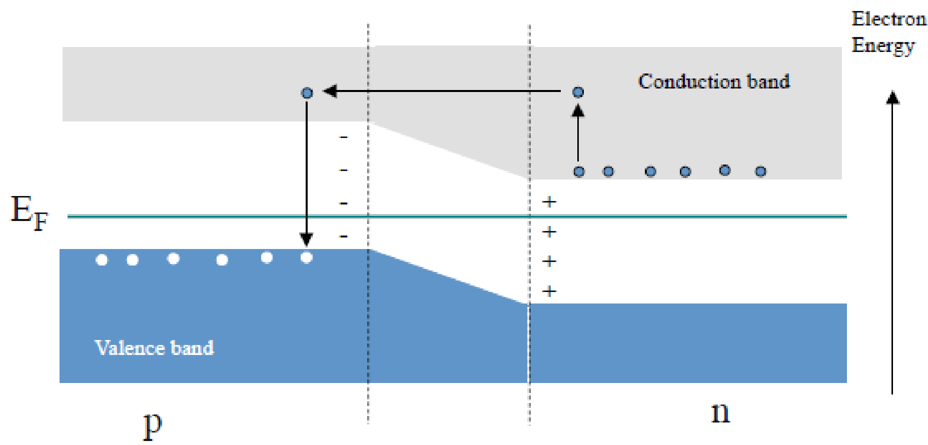
$$j = j_p^{(n)}(x) + j_n^{(p)}(x) = e\left(\frac{D_p}{L_p}p_n + \frac{eD_n}{L_n}n_p\right)\left(e^{\frac{eU}{kT}} - 1\right)$$

$$j = j_0\left(e^{\frac{eU}{kT}} - 1\right)$$

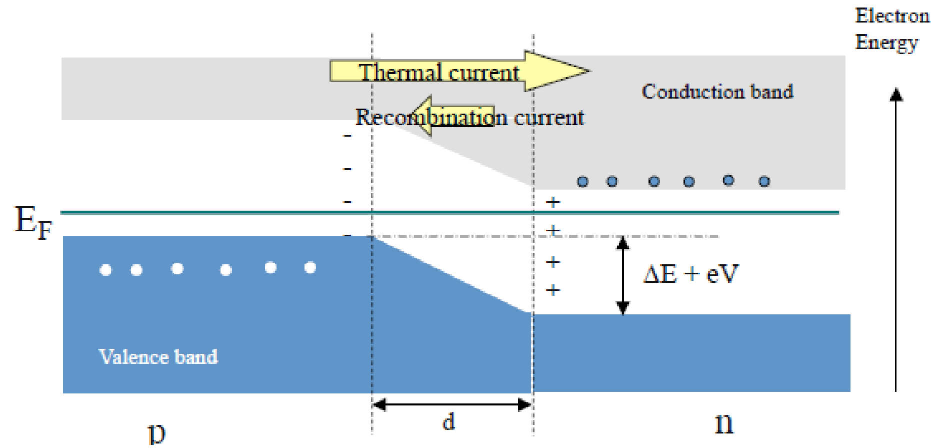


+I (mA)
Forward Current
Forward Bias
"knee"
Reverse Breakdown Voltage
-V
+V
Reverse Voltage
Forward Voltage
-50mA Germanium
-20mA Silicon
0.3v Germanium
0.7v Silicon
"Zener" Breakdown or Avalanche Region
Reverse Bias
-I (μA)
Reverse Current

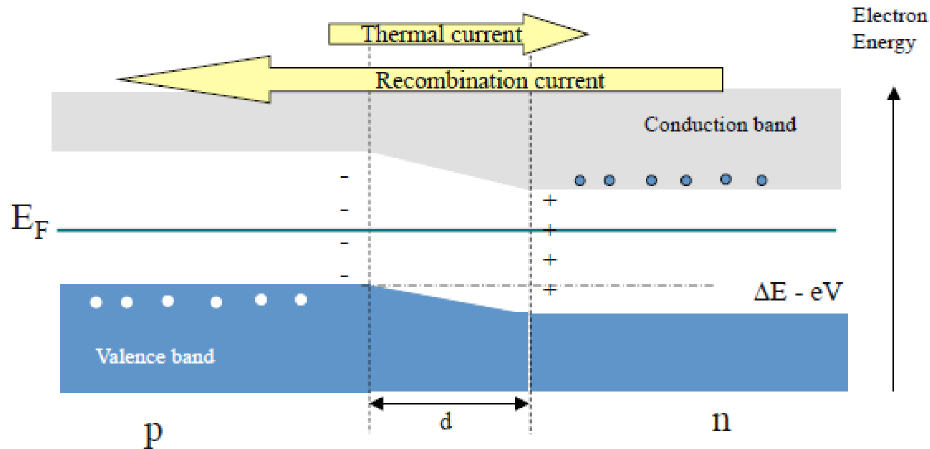
Potential offset



Equilibrium



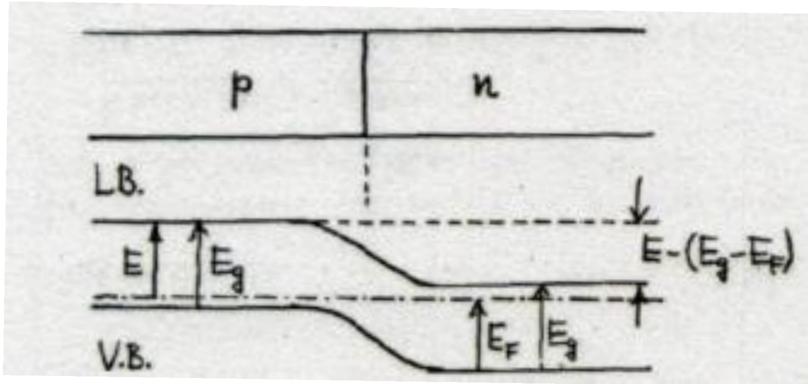
Reverse bias



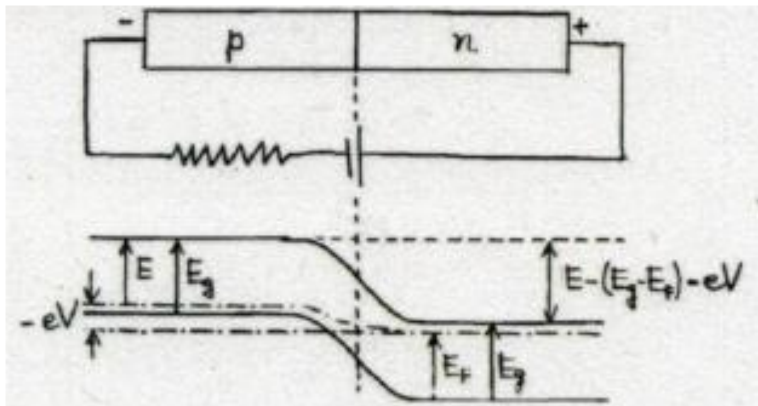
Foreward bias

http://faculty.cord.edu/luther/physics225/Handouts/semiconductors_handout.pdf

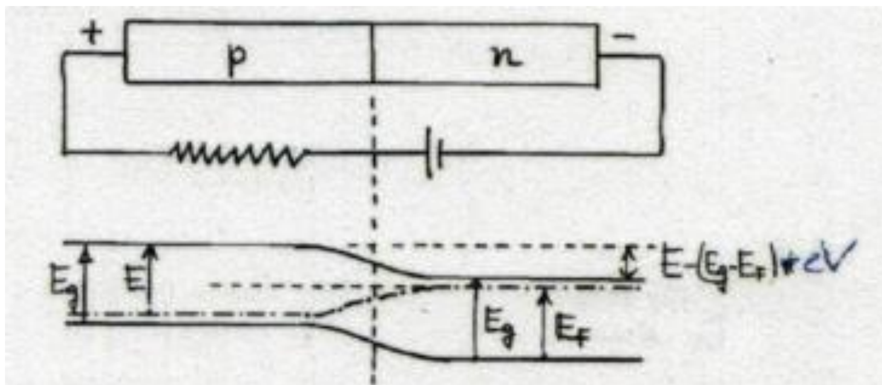
Potential offset



Equilibrium



Reverse bias



Foreward bias

J. Peisl, LMU 1990

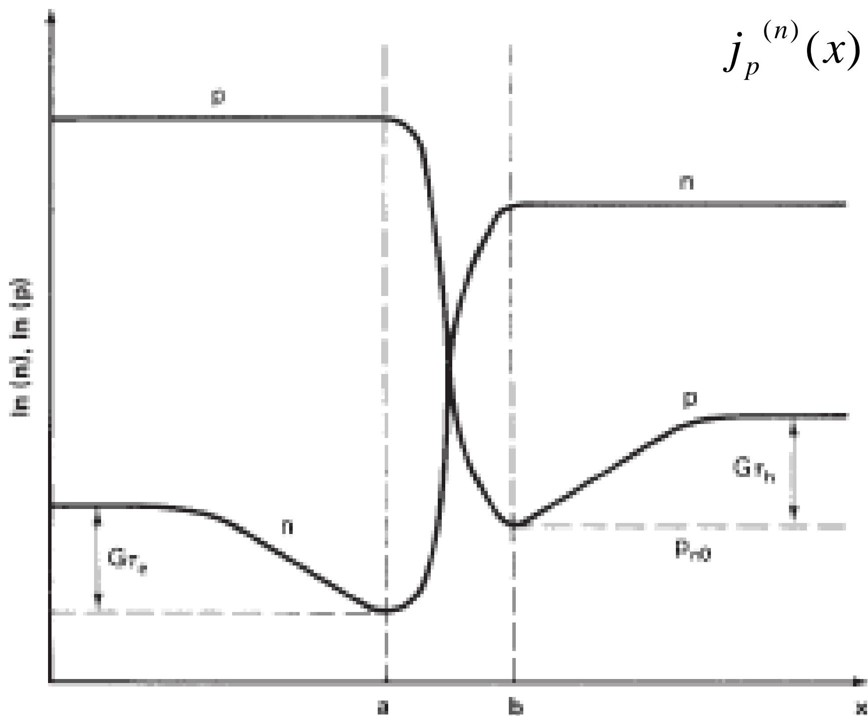
pn-junction under illumination

Generation of carriers by light

$$\frac{d^2p}{dx^2} = \frac{p-p_n}{L_p^2} - \frac{G}{D_p}; \qquad \frac{d^2n}{dx^2} = \frac{n-n_p}{L_n^2} - \frac{G}{D_n}$$

$$p(x) = p_n + G\tau_e + [p_n(e^{eU/kT} - 1) - G\tau_h]e^{(x_n - x)/L_p}$$

$$j_p^{(n)}(x) = e\frac{D_p}{L_p}p_n[e^{eU/kT} - 1]e^{(x_n-x)/L_p} - eGL_n e^{(x_n-x)/L_p}$$



$$j = j_0(e^{eU/kT} - 1) - j_L$$

→

$$I = I_s\left[\exp\left(\frac{eU}{kT}\right) - 1\right] - I_L$$

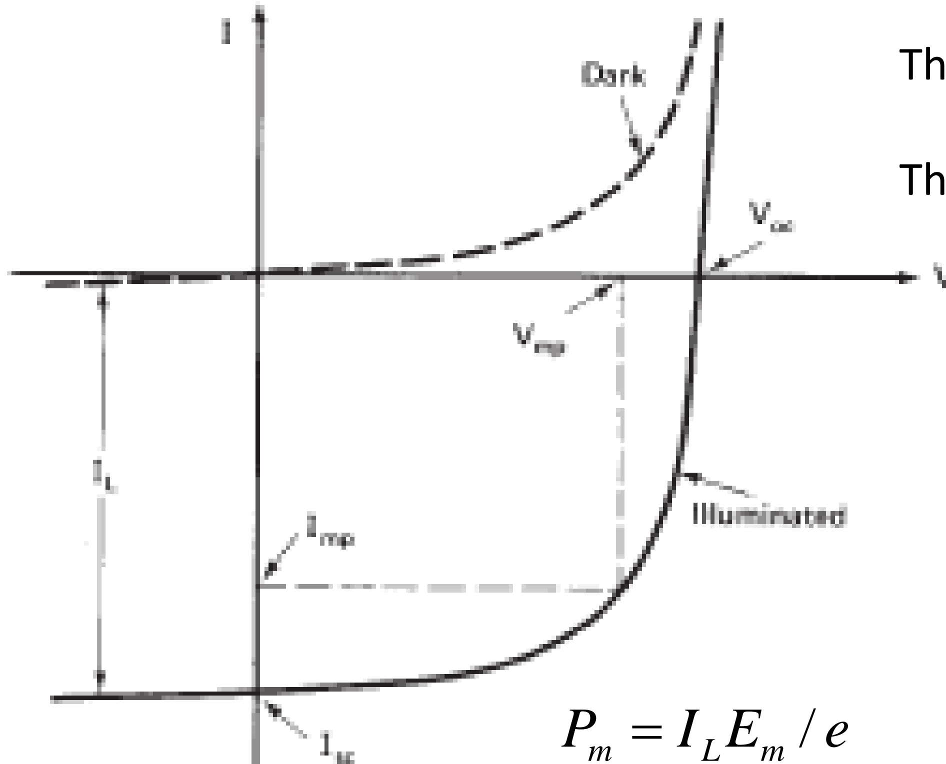
$$I_L = eAG(L_n + L_p + W)$$

I_L - Leakage current

Solar cells

Solar cell operates between -1 < I < 0A and 0 < V < 1V.
open-circuit voltage V_{oc} is for I=0. At V=0 the $I=I_L$ is the short-circuit current.
Only the rectangle I_{sc} x V_{oc} can be used for power conversion; load resistance R_L sets the working point at I_m and V_m defining the filling factor .

$$FF = \frac{I_m V_m}{I_{sc} V_{oc}} < 1$$



The open circuit voltage: $V_{oc} = \frac{KT}{e}\ln(\frac{I_L}{I_s}+1)$

The output power is

$$P = IV = I_s V\left[\exp\left(\frac{eV}{kT}\right)-1\right]-I_L V$$

The condition dP/dV=0 defines the working point for V_m

$$V_m = V_{oc} - \frac{kT}{e}\ln(1+\frac{kT}{eV_m})$$

and approximate for I_m $I_m \approx I_L(1-\frac{kT}{eV_m})$

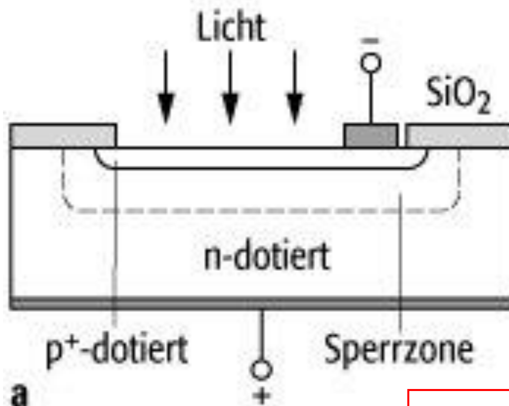
Energy delivered per photon at R_L

$$E_m = e\left[V_{oc} - \frac{kT}{e}\ln(1+\frac{kT}{eV_m}) - \frac{kT}{e}\right]$$

$$P_m = I_L E_m / e$$

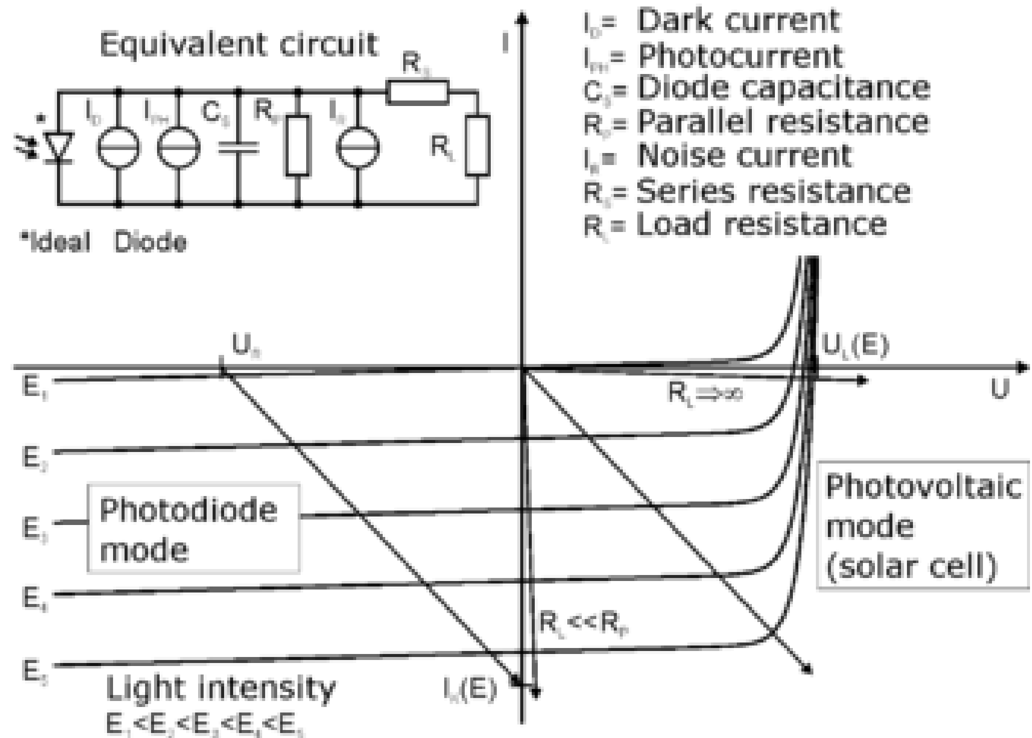
power conversion efficiency $\eta = \frac{P_m}{P_{in}}$

Photo -diode



Because of $w \propto \sqrt{\frac{U}{N}}$

Very pure Si material



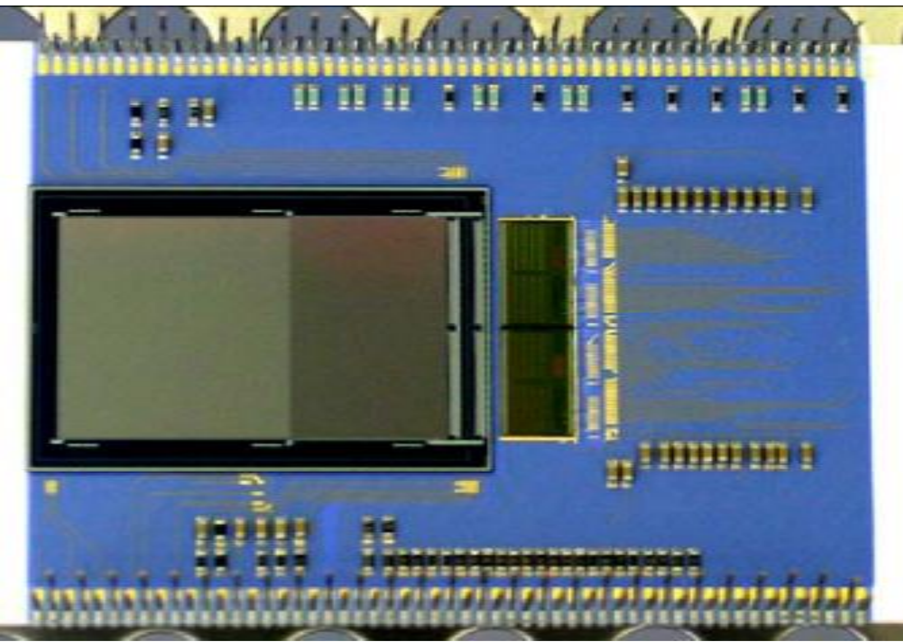
Creation of e-h – pairs within the space charge region, because of reverse bias, e and h become separated towards external contacts. Number of created e-h pairs depends on photon energy: E_g(k=0) – Si – 3.56eV

$$N = h\nu / E_g(k=0)$$

hν=10keV, N = 2808

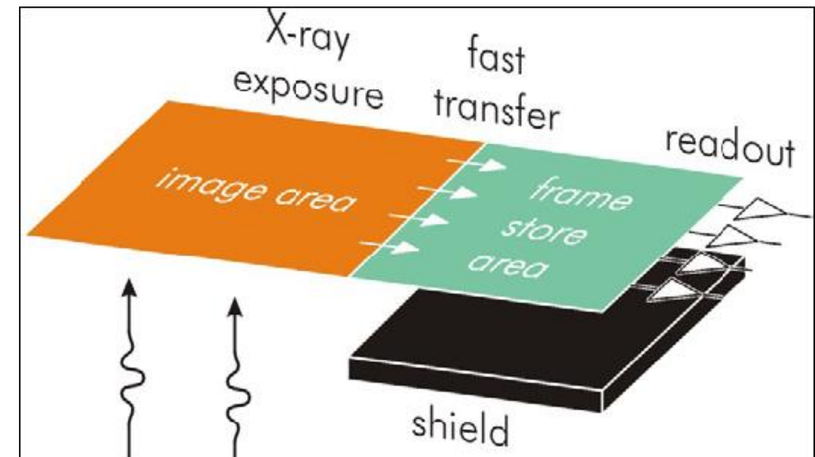
Pn-CCD

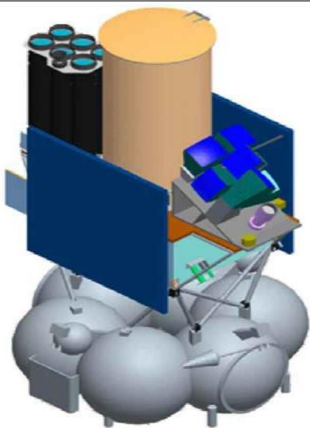
Energy dispersive pixel detector for X-rays



Alternative approach

Project has started June2007

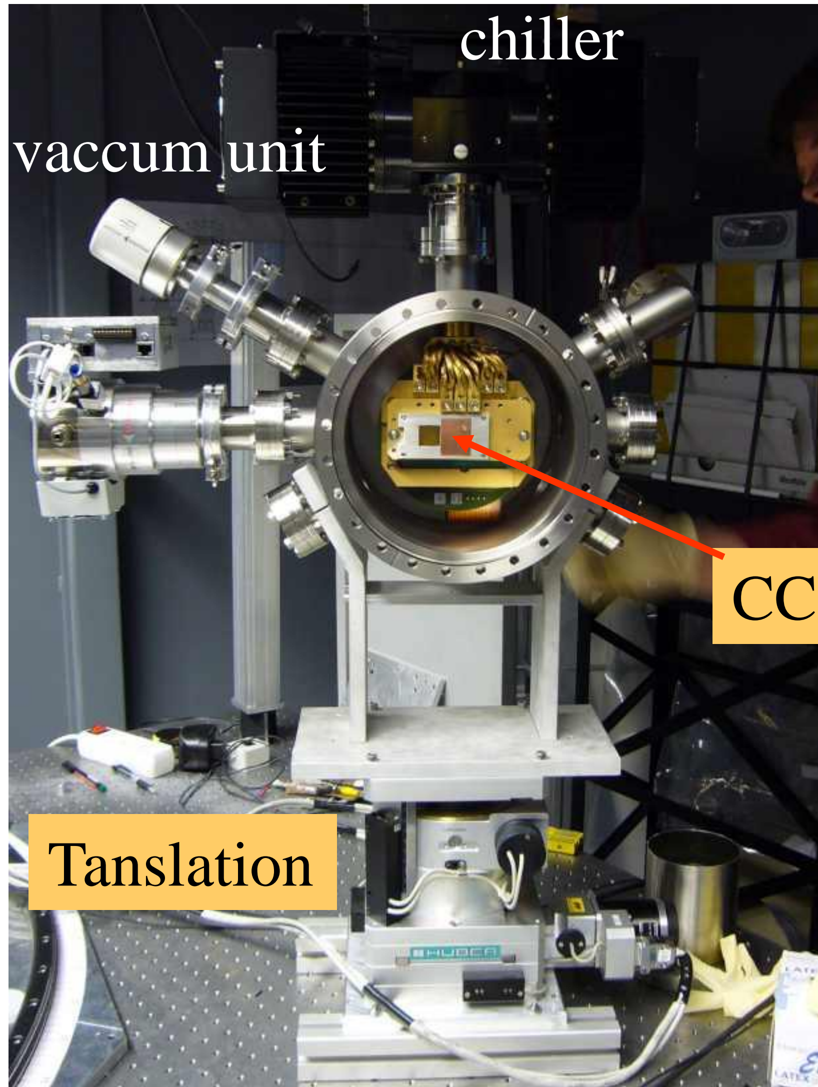


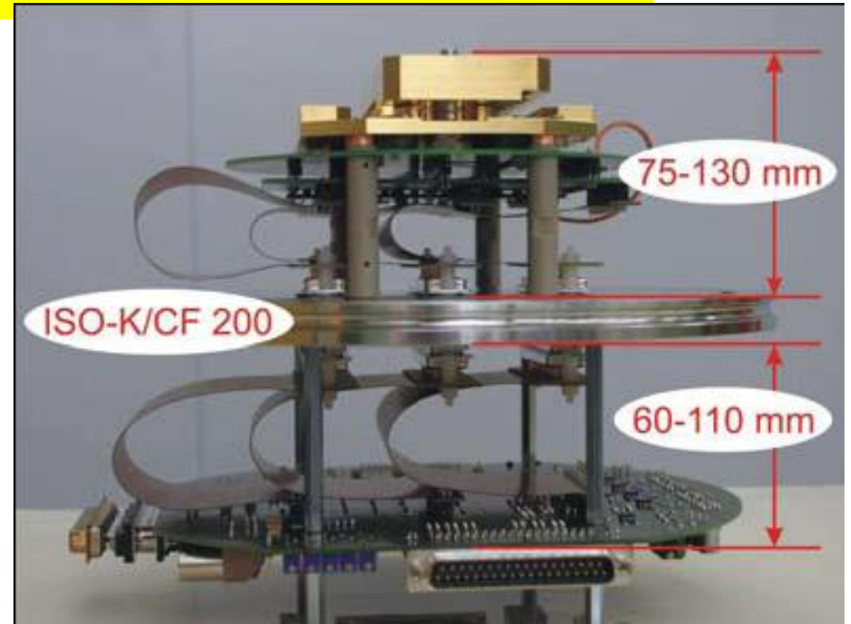


pn-CCD X-ray detector type of MPI HLL Munich, originally *developed for* XMM-Newton Satellite mission (ESA).
Since launch 1999 excellent spectroscopy and imaging,

Our Project : Application of pnCCD for use of Synchrotron Radiation

General setup of the detector





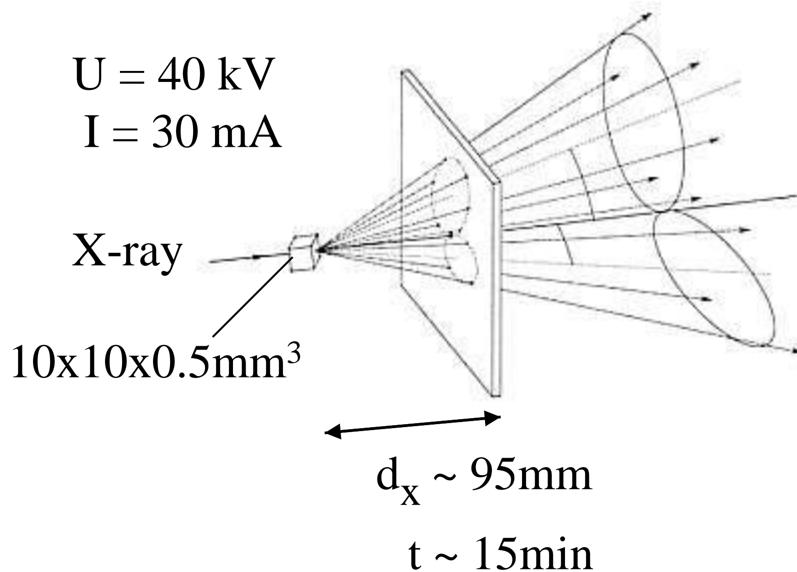
CCD-chip

Application: Laue diffraction at Lithium aluminate ($LiAlO_2$ [100])

→ tetragonal structure

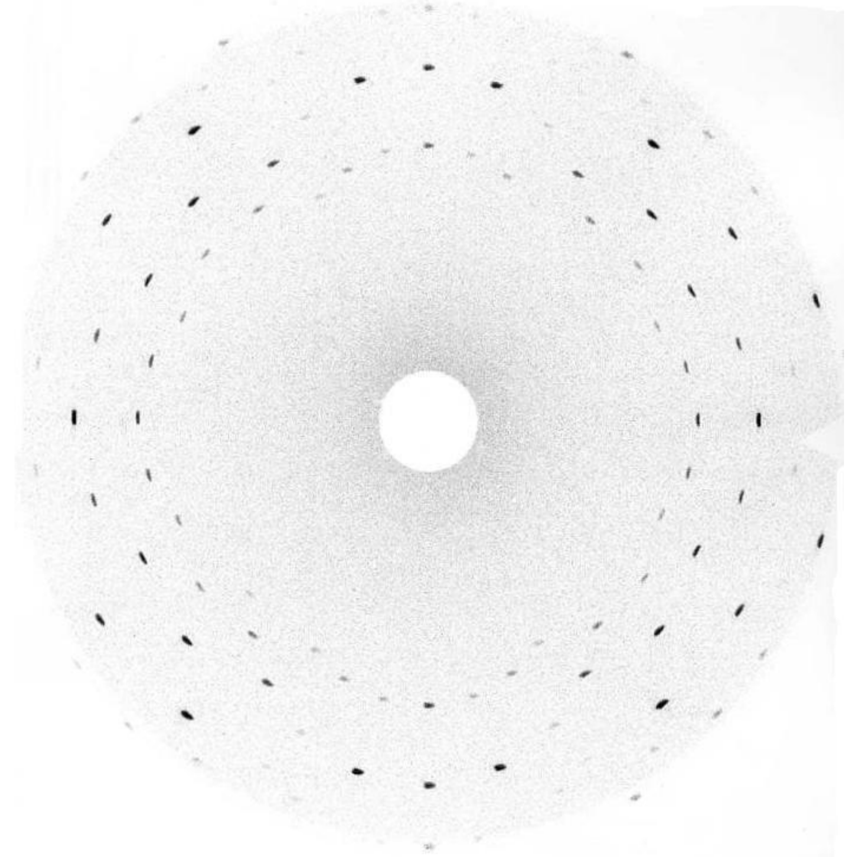
$a = 5.1687$ Å
$c = 6.2676$ Å

Illumination parallel 100



d_x ~ 95mm

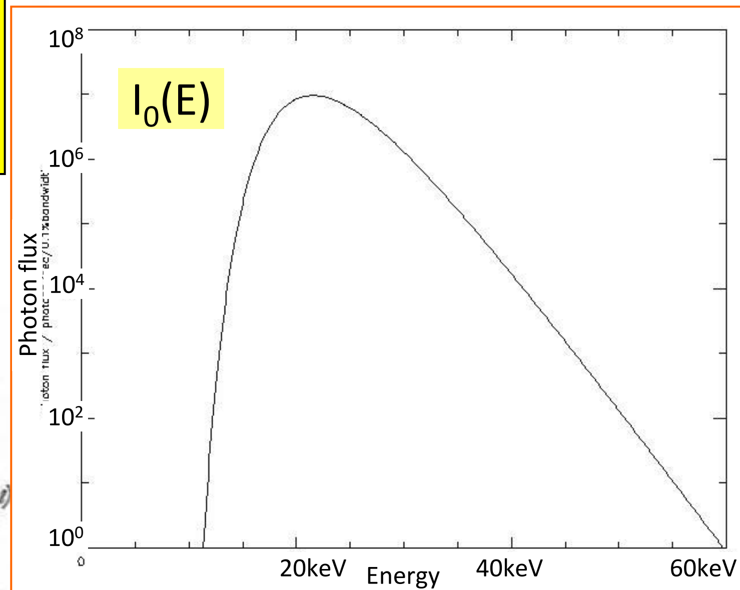
t ~ 15min



2-fold symmetry!

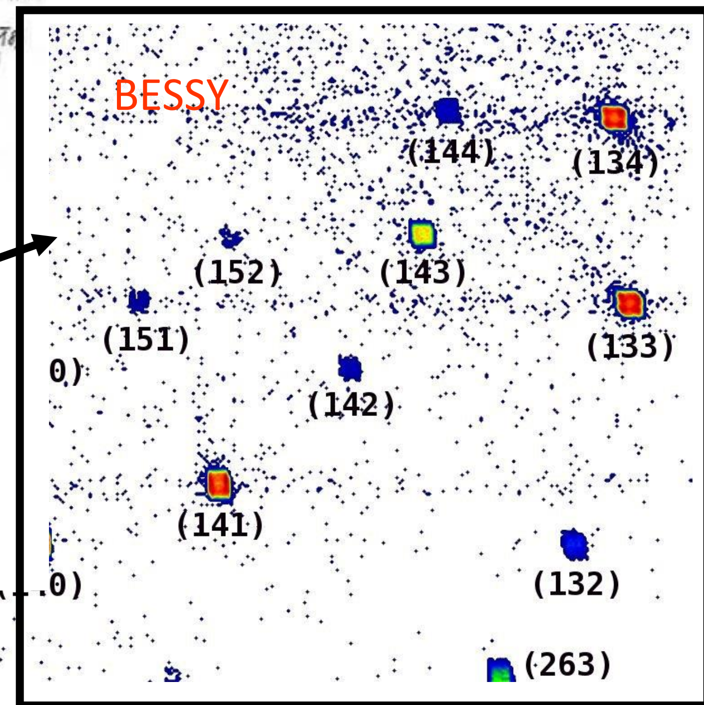
Measurement with pnCCD In May2008

home



2θ

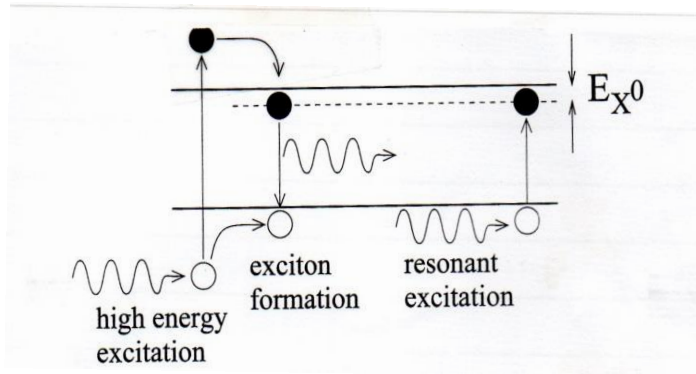
E

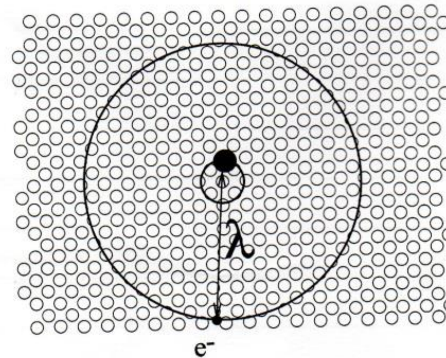


Measure Bragg angle and energy simulataneous

$E=hc/(d \sin\theta)$

Excitons in semiconductors





Excitons are created once a photon is absorbed by the semiconductor exciting an electron from VB into CB creating h+ in VB and e- in CB connected via Coulomb interaction.

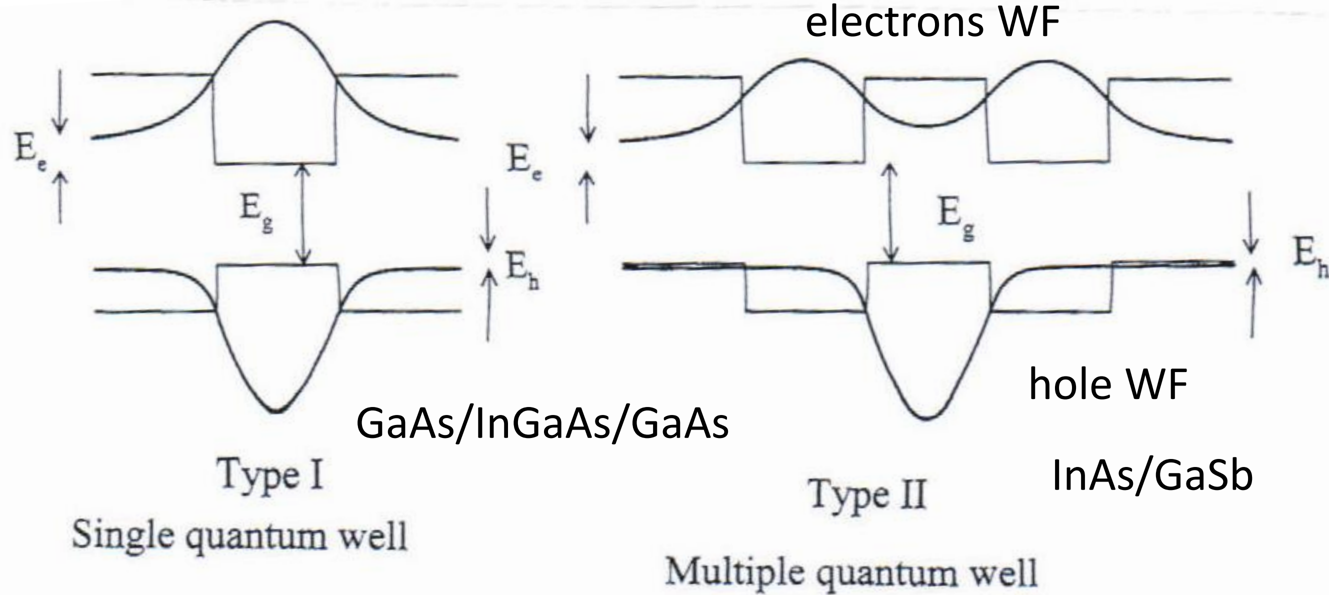
→ bound electron-hole pair = "exciton", is quite stable and can have a long life time, of order of nanoseconds.

Exciton binding energy $E_X = 13.6eV \frac{\mu}{\varepsilon_r{}^2} \frac{1}{n^2}$ exciton radius $\lambda_x = 0.0529nm \frac{\varepsilon_r}{\mu} n^2$

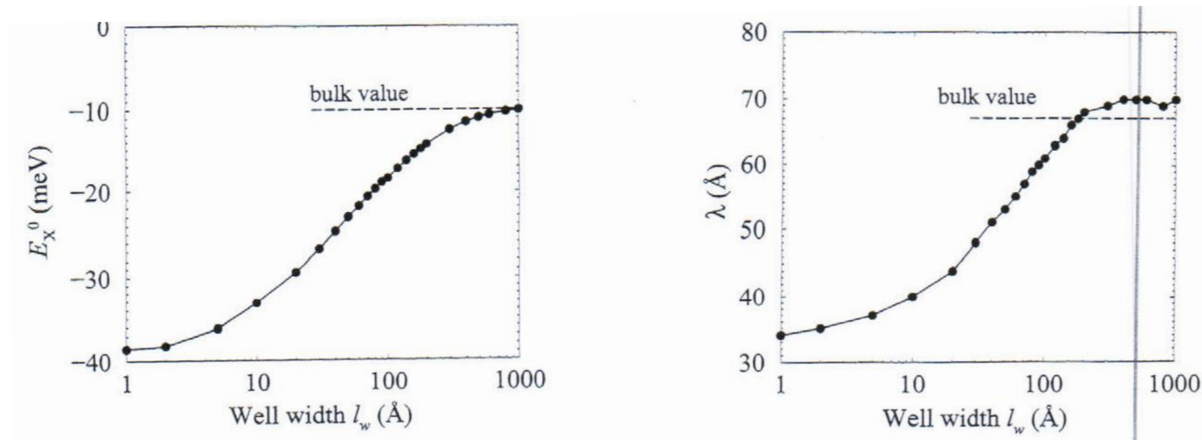
Considering reduced mass $\frac{1}{\mu} = \frac{1}{m_e^*} + \frac{1}{m_h^*}$

In case of GaAs m^*_e=0.067m and m_{hh}^*= 0.62m → μ=0.060m. Using e_r =13.18 the exciton binding energy is E_x=-4.7 meV and the Bohr radius λ=11.5nm.

Excitons in heterostructures



Thin well structures



$$\lim_{l_w \to \infty} E_{X^0} = E_{X^0}^{3D} \quad \text{and} \quad \lim_{l_w \to 0} E_{X^0} = 4E_{X^0}^{3D}$$

pn semiconductor diode (LED)

Super luminescent LED (SLD)

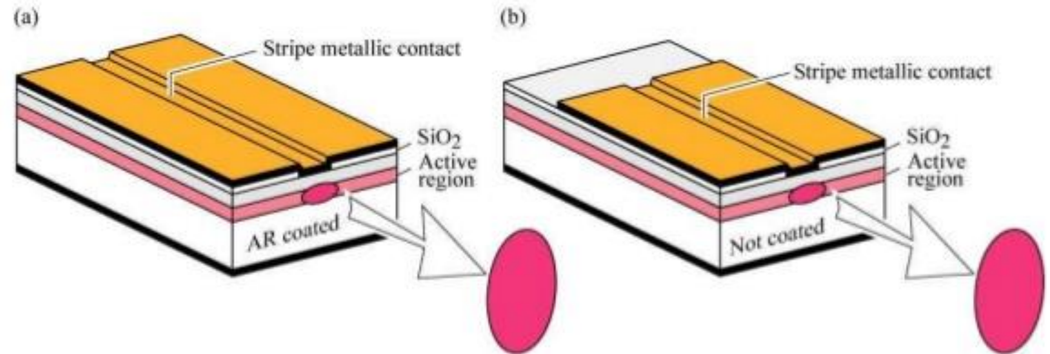
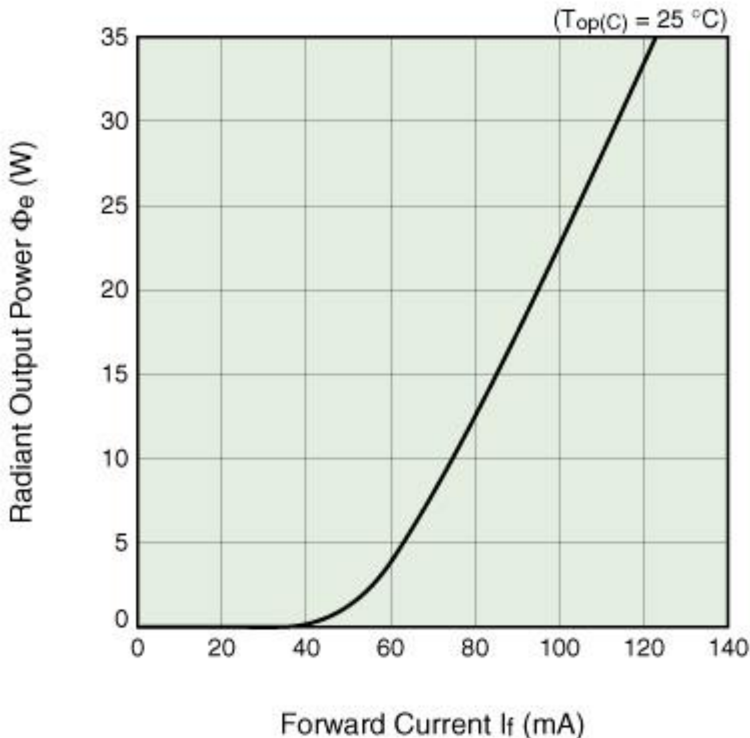


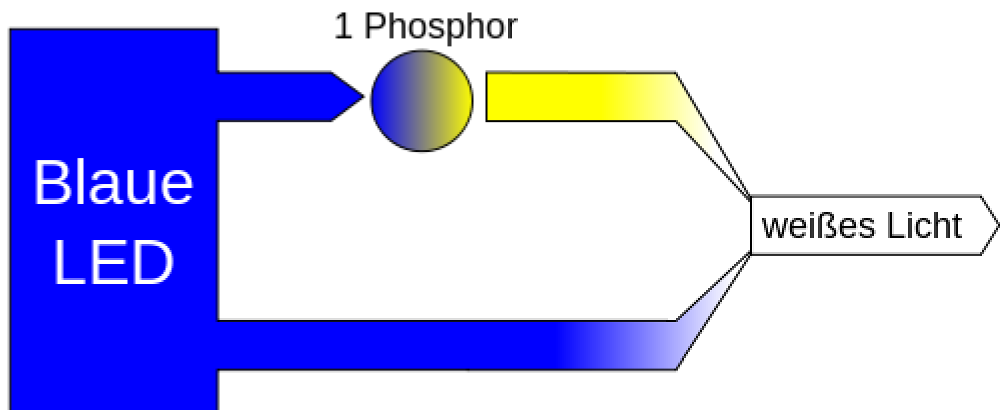
Fig. 23.9. Common structures of superluminescent diodes (SLDs). (a) SLD with cleaved facets coated with anti-reflection (AR) coatings. (b) SLD with cleaved, reflecting facets and stripe contact injecting current over the partial length of the device.

E. F. Schubert
Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org



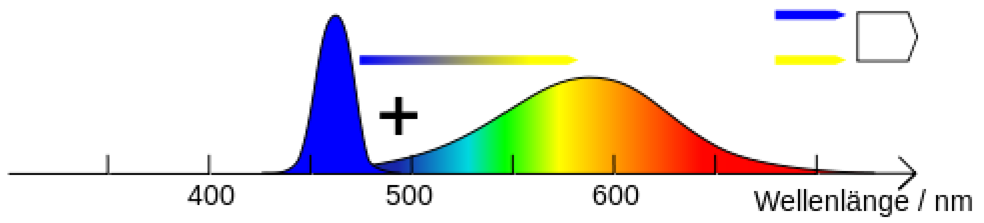
Radiant flux 30mW at operating voltage 1.8V

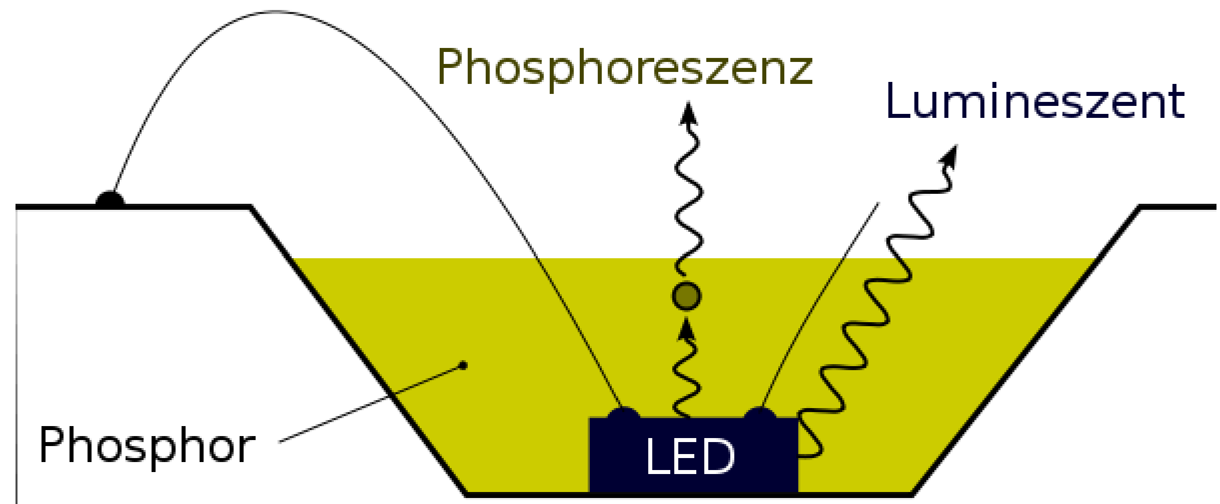
Hamamatsu GmbH



AlGaN - λ= 400nm

Withe LED

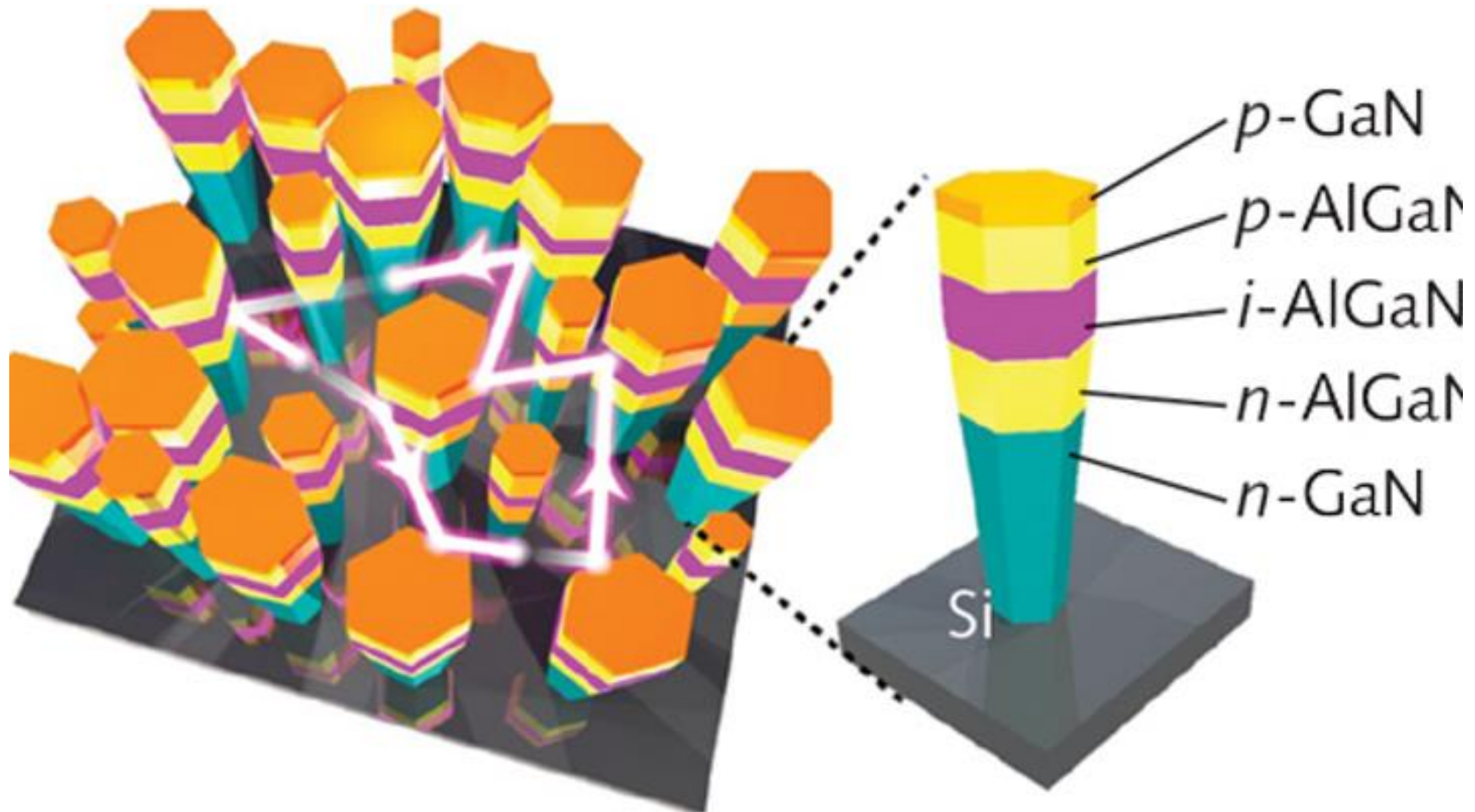




Wikipedia : white LED

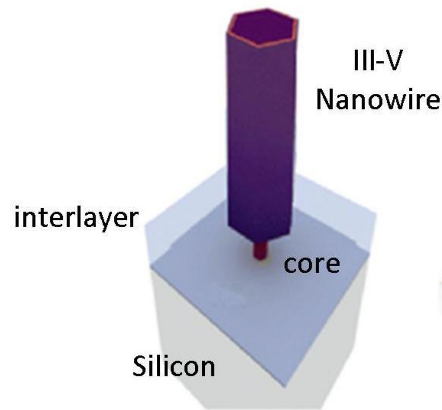
Laser Diodes: AlGaN nanowire laser diode emits at 239 nm

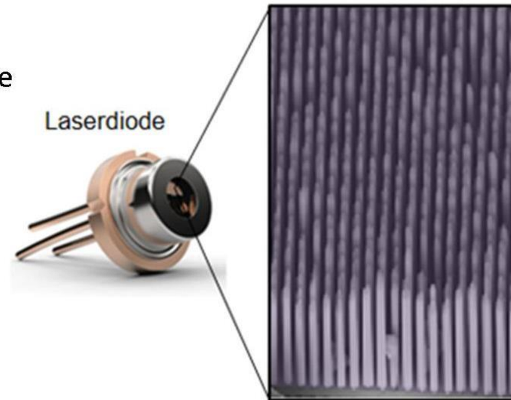
Over the years, the short-wavelength limit of laser diodes has moved from the red end of the visible spectrum to the near-UV.

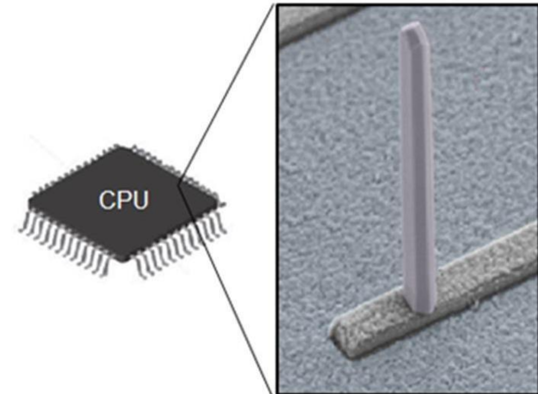


https://www.laserfocusworld.com/lasers-sources/article/16546978/

MONOLITHICALLY INTEGRATED NANOWIRE LASERS ON SILICON







(a) Patented Nanowire laser geometry. (b) Vertically emitting nanowire laser array in a laser diode application. (c) Nanowire laser coupled to a silicon waveguide for on-chip and chip-to-chip data communication.

https://www.baypat.de/technology-offers/monolithically-integrated-nanowire-lasers-iii-v-lasers-on-silicon