Elastic properties of solids

Elastic properties of solids are described by Hook´s law $\sigma = E\varepsilon$; where σ is the applied stress [N/cm²] and εthe resulting strain [Δa/a]. Besides the elastic modulus (or Young's modulus) E, additional parameters characterizing a deformation are: Poisson ratio $\nu = (\Delta d/d)/(\Delta l/l)$,the ratio between lateral contraction and relative expansion in the direction of stress, and shear stress $\sigma = \mu\Theta$, where μ is the shear constant and Θ the shear deformation angle. For an isotropic solid, E and ν determine the relative change in volume under deformation: $\frac{\Delta V}{V} = \frac{\sigma}{E}(1-2\nu)$. Deformation on hydrostatic pressure is given by the bulk modulus (module of compression) $K = E/[3(1-2\nu)]$.

However, crystalline solids are anisotropic. Therefore, both σ and ε are second rank tensors, containing up to 6 independent coefficients (one in cubic, 2 in tetragonal, 3 in hexagonal system). Subsequently Young's modulus is a forth rank tensor with up to 27 independent elements (3 in cubic system: c_{11}, c_{12}, c_{44}). Considering Voigt´s notation, Hook´s law transforms into

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} --> \sigma_i = c_{ij}\varepsilon_j .$$

The elastic modules differ for various metallic, covalent and ionic compounds. Calculated elastic constants do not reproduce the experiment in all cases. Prominent example is the anisotropy ratio in cubic materials: $A = 2c_{44}/(c_{11}-c_{12})$ deviating from unity which is a measure for the fact that elasticity in solids cannot be described by summing up individual next-neighbour interactions. Elastic modules show both a temperature and pressure dependence.

Elastic modules can be determined from Phonon dispersion, Brillouin scattering and other methods. Most precise determination is achieved by a measurement of the sound velocity after ultrasonic exposure. Data evaluation is based on solution of the wave equation in solids: $\rho \ddot{u} = div\sigma$; where ρ is the density and u the relative displacement. Note, u and σ are tensor quantities. Assuming an ultrasonic wave as a plane wave propagating as longitudinal wave along [100] in a cubic crystal results in a sound velocity $v_{long}[100] = \sqrt{\frac{c_{11}}{\rho}}$, for a transversal wave along [100] one gets: $v_{trans}[100] = \sqrt{\frac{c_{44}}{\rho}}$; for waves propagating along [110] one obtains one longitudinal $v_{long}[110] = \sqrt{\frac{c_{11}+c_{12}+2c_{44}}{\rho}}$ and two transverse $v_{trans\_1}[110] = \sqrt{\frac{c_{11}-c_{12}}{\rho}}$, $v_{trans-2}[110] = \sqrt{\frac{c_{44}}{\rho}}$ velocities.

The bulk modulus in cubic system is $K = \frac{c_{11}+2c_{12}}{3}$ and Poisson's ratio $\nu = \frac{c_{12}}{c_{11}+c_{12}}$.